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# Summary Report on Permanent Deformation in Asphalt Concrete

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# Abstract

This report evaluates information on the permanent deformation characteristics of asphalt-aggregate mixtures, with an emphasis on laboratory test techniques for measuring mixture resistance to permanent deformation, and on methods for prediction of permanent deformation (rutting) in the upper asphalt-bound layers of pavement structures.

Factors influencing the amount of rutting are summarized; the influence of mixture density and aggregate structure emphasize the importance of proper preparation of test specimens to duplicate in situ conditions.

The laboratory test methods associated with the predictive methodologies that are evaluated include: a) uniaxial and creep tests; (b) uniaxial and triaxial repeated load test; c) triaxial dynamic tests; d) diametral tests, creep and repeated load tests; e) hollow cylinder tests, combined axial and torsional loading; f) simple shear tests, unconfined and confined; and g) wheel-track tests.

From these evaluations, specific test methodologies, either existing or new, that have the potential to define the propensity for rutting of dense-graded paving mixtures, are ranked in order of preference.

# Executive Summary

Permanent deformation (rutting) of asphalt pavements has a major impact on pavement performance. Rutting reduces the useful service life of the pavement and, by affecting vehicle handling characteristics, creates serious hazards for highway users. Highway materials engineers have been handicapped in their efforts to provide rutting resistant materials in that existing methods for testing and evaluating asphalt-aggregate mixes are empirical and do not give a reliable indication of in-service performance.

The purpose of this summary report is to evaluate available information concerning the permanent-deformation characteristics of asphalt-aggregate mixtures with particular emphasis on 1) laboratory test techniques for measuring the resistance of these mixtures to permanent deformation and 2) methodologies that permit prediction of the amount of permanent deformation (rutting) in the upper asphalt-bound layers of the pavement structure.

Although mixture densification (volume change) has some effect, rutting is principally caused by repetitive shear deformations under traffic loading. Among the factors influencing the amount of rutting are the magnitudes and pressures of tire loads, the volume of traffic, the thermal environment, and various mixture properties. Among the influential mixture properties, aggregate characteristics (specifically rough surface texture, angularity, and dense gradation) are particularly important contributors to permanent-deformation resistance. The amount and stiffness of the asphalt or modified asphalt binder are also important, with lower asphalt contents and stiffer binders providing improved resistance to permanent deformation. The significant influence of mixture density and aggregate structure underscores the importance of duplicating, in laboratory-compacted specimens, conditions expected under field compaction and traffic loading.

Layer-strain and viscoelastic methodologies are currently used to predict the development of permanent deformation in asphalt-bound layers. These methodologies, together with associated laboratory test methods, are briefly summarized herein. The

test methods include: 1) uniaxial and triaxial creep tests; 2) uniaxial and triaxial repeated load tests; 3) triaxial dynamic tests, 4) diametral tests, creep and repeated load; 5) hollow cylinder tests, combined axial and torsional loading; 6) simple shear tests; and 7) wheel-track tests.

The literature sources reviewed for this report suggest that 1) neither the layer-strain procedure nor conventional viscoelastic analysis has been able to accurately and reliably model pavement rutting; and 2) while it is recognized that repetitively applied shear stresses from tire loading are largely responsible for rutting, laboratory test procedures in common use do not properly incorporate such stress states.

Based on this study, SHRP has concluded that the most promising test methods for further evaluation include, in order of preference, the following: 1) simple shear, creep and repeated loading; 2) triaxial compression, creep and repeated loading; and 3) uniaxial compression, creep and repeated loading. Each of these tests has potential for use both in mixture design systems and in analysis systems that predict the development of permanent deformation under specific traffic and environmental conditions. The diametral test does not provide a suitable measure of resistance to permanent deformation because its complex stress state confounds the meaningful interpretation of test results. While hollow-cylinder and wheel-track testing simulate field conditions, their complexity makes them unsuitable for routine use. However, both are very useful for special investigations, including the evaluation and validation of simpler test systems.

# 1

## Introduction

### 1.1 Problem Definition

A major concern today in many parts of the United States is excessive permanent deformation (rutting) in heavy duty asphalt-concrete pavements resulting from frequent repetitions of heavy axle loads, many of which are operating with radial tires having pressures 20 to 25 psi higher than the bias-ply tires which they have replaced (e.g., 105 psi versus 80 psi). Rutting gradually develops with increasing numbers of load applications and appears as longitudinal depressions in the wheel paths.

These depressions or ruts are of concern for at least two reasons: 1) if the surface is impervious, the ruts trap water and, at depths of about 0.2 in., hydroplaning (particularly for passenger cars) is a definite threat; and 2) as the ruts progress in depth, steering becomes increasingly difficult, leading to added safety concerns. Accordingly, it is important that a test procedure be developed which will reasonably predict the propensity for an asphalt paving mixture to develop excessive permanent deformation under repeated loading by heavy traffic.

### 1.2 Purpose

The purpose of this report is to present an evaluation of available information about the permanent-deformation characteristics of asphalt concrete. Particular emphasis is placed on methodologies which permit prediction of the amount of rutting which develops in asphalt-bound layers under the repetitive action of traffic and on associated test techniques for those materials. From these evaluations, specific test methodologies,

either existing or new, that have the potential to define the propensity for rutting of dense-graded paving mixtures are identified.

### 1.3 Objectives

The objectives of this study are to:

- 1) Describe the mechanism of rutting and review the factors influencing the permanent-deformation characteristics of asphalt paving mixtures;
- 2) Evaluate current analytical procedures for predicting permanent deformation in asphalt-bound layers;
- 3) Critically examine test methods which are utilized to assess the resistance of asphalt mixtures to permanent deformation. These methods include: a) uniaxial and triaxial creep tests; b) uniaxial and triaxial repeated load tests; c) triaxial dynamic tests; d) diametral tests, creep and repeated load; e) hollow cylinder tests, combined axial and torsional loading; f) simple shear tests, unconfined and confined; and g) wheel-track tests; and
- 4) List, in order of preference, methods for measuring the permanent-deformation characteristics of asphalt concrete.

The following briefly describes the organization of this summary report. Background information, including a historical perspective, a discussion of the mechanism of rutting, a brief discussion of the factors affecting rutting, and current criteria to assess the severity of rutting in situ, is included in Section 2. Section 3 includes a categorization of methods used to predict rutting in asphalt-bound layers and a summary of some of the research endeavors associated with these predictive methodologies. Section 4 provides a summary of a number of methods to measure the permanent-deformation characteristics of asphalt mixtures, while Section 5 ranks these methods according to their suitability for routine laboratory use.

Discussion is directed in Section 6 to the importance of shear stresses in developing rutting in the upper portions of asphalt-concrete layers, and attention is called to the fact that none of the current analytical procedures permits a proper evaluation of the influence of these stresses on estimates of permanent deformation. Section 7 provides conclusions based on this detailed evaluation together with general recommendations for both an analytical study and a test program to permit the development of an improved methodology for mitigating rutting in asphalt-concrete pavement layers. Appendix A includes a description of the test program developed on the basis of this evaluation.

# 2

## Background

### 2.1 Historical Perspective

When considering rutting in asphalt-concrete pavements, two facets must be considered. One is associated with *asphalt mixture design* and the other with *rutting prediction* in the pavement structure. Until recently these have usually been considered separately, thereby largely excluding expected traffic and environmental exposure from consideration in the mixture design process.

Relative to mixture design, two methods are used in the United States by highway authorities to select the proper amount of binder. One is based on the Marshall test and the other on the Hveem Stabilometer. Thirty-eight states use the Marshall test, and 11 states, the Hveem Stabilometer (Kandhal and Koehler, 1985). With recent increases in traffic--both in terms of repetitions and heavier axle loads as well as increases in tire pressures--increased rutting has been observed, particularly in states that use the Marshall methodology (Federal Highway Administration, 1987).

In both Marshall and Hveem methods, criteria for mixture design are based on past correlations of laboratory test results with field performance. Unfortunately, the conditions under which the criteria were developed have changed in recent years, leading at times to mixtures which exhibit rutting. Thus, there is increased interest in developing mixture evaluation procedures which will be more responsive to changing traffic conditions, e.g., the National Cooperative Highway Research Program (NCHRP) Project 9-6 and referred to as the AAMAS Project<sup>1</sup> (Von Quintus et al., 1988).

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<sup>1</sup>AAMAS refers to Asphalt Aggregate Mixture Analysis System.

With regard to rutting prediction, little formal work had been done prior to 1960. Methods of pavement design in use in the United States at the time, e.g., the California Bearing Ratio (CBR) procedure and the "R" value methodology, utilized empirical measures of the shear resistance of pavement components. By insuring an adequate thickness above a material with a specific CBR or "R" value, rutting in the pavement structure presumably would be limited to a tolerable amount for the duration of the design period. This minimum thickness was also dependent both on the magnitudes and numbers of repetitions of the traffic loads.

At the First International Conference on the Structural Design of Asphalt Pavements in 1962, the Shell Oil Company (Dorman, 1962) presented the first pavement design approach which explicitly considered both fatigue and rutting as mechanisms of distress. As a part of this design method, rutting was controlled by limiting the vertical compressive strain at the top of the subgrade. This approach, together with conventional asphalt-concrete mixture design, appeared to be suitable in mitigating pavement rutting for a number of subsequent years. During this same period, fatigue distress also became less prevalent due to a trend toward thicker asphalt surfaces and larger asphalt contents. With these changes, however, and with gradual increases in allowable axle loads, tire pressures, and truck volumes, the need for an improved approach to predicting rutting, particularly in the upper 10 to 12 inches of the pavement, became apparent.

At the Third International Conference (1972) on the Structural Design of Asphalt Pavements, methodology was presented for predicting rutting in flexible pavements. Barksdale (1972) and Romain (1972) described one such approach, identified herein as the layer-strain methodology. Since these early efforts, considerable additional work has been undertaken including subsequent verification of rutting predictions by comparisons with observations from test tracks or roadways. These design approaches utilize "fundamental properties" of the materials in the pavement section and account in different ways for the effects of variations in loading and environmental conditions (e.g., moisture and temperature) on the amount of rutting.

A 1976 symposium on rutting in asphalt pavements, sponsored by the Transportation Research Board, included several papers that emphasized rut-depth predictions and the test procedures which could be used to define asphalt mixture characteristics necessary for the prediction of permanent deformation.

At the Fourth International Conference on the Structural Design of Asphalt Pavements (1977), additional design methods were described which considered rutting. The procedures, encompassing methodologies both to limit surface rutting to some prescribed

level and to predict rutting in either the asphalt-bound layer or the entire pavement structure, were of the following types:

- 1) Statistical techniques based on observed rutting performance;
- 2) Limiting subgrade strain procedures;
- 3) Elastic analysis together with creep test data; and
- 4) Linear viscoelastic analyses.

The sophistication of these methods varied widely. Finn et al. (1977) used stepwise regression techniques to relate rutting in both conventional and full-depth pavements to stress, surface deflection, and number of load repetitions. Claessen et al. (1977) modified the original Shell subgrade strain criteria slightly. In the revised methodology, the final design is checked for rutting in the bituminous layers using the creep test approach developed by Hills (1974) and van de Loo (1974). Kenis (1977) related plastic strain to stress, temperature, loading time, and moisture content and used viscoelastic theory to compute rut depth.

At the Fifth International Conference on the Structural Design of Asphalt Pavements (1982), little new information was presented on rutting prediction. Design methodologies that were described relied primarily on limiting the vertical compressive strain in the subgrade. Moreover, only a limited amount of work was presented which dealt specifically with verification of these methods. It should also be noted that only one of six sessions dealt with materials characterization, and this session offered little new information on either test methods or prediction procedures. One might thus conclude that, at this time, the pavement engineer's interest in rutting was on the decline.

Following the Fifth Conference in 1982, however, interest in the rutting problem has been renewed. A continuous series of papers has been published regarding the occurrence and prediction of rutting in flexible pavements, together with information concerning wheel-load and tire-pressure effects on stress and strain within the pavement layers. Research has been directed both to characterizing materials and to developing models for predicting rutting of asphalt pavements. Reported methodologies range in sophistication from empirical (e.g., Uzan and Lytton, 1982) to the use of viscoelastic-plastic considerations (e.g., Abdulshafi, 1983).

At the Sixth International Conference (1987), papers by Eckmann and by Eisenmann and Hilmer are of particular interest. Eckmann's study combined dynamic creep testing with layer-strain analysis to predict rutting in full-scale test pavements. His prediction models showed good agreement with actual field measurements. Eisenmann and Hilmer studied the influences of wheel loading and tire pressure on the magnitude of the rutting

in asphalt pavements. Full-scale tests were performed using various wheel loads, inflation pressures, and wheel arrangements. The development of rutting was measured directly, and the effects of the varying test conditions were analyzed using regression analysis.

While this historical summary has concentrated on developments presented at the International Conferences on the Structural Design of Asphalt Pavements, it mirrors developments presented in other literature including that of The Association of Asphalt Paving Technologists and the Transportation Research Board.

## 2.2 Mechanism of Rutting

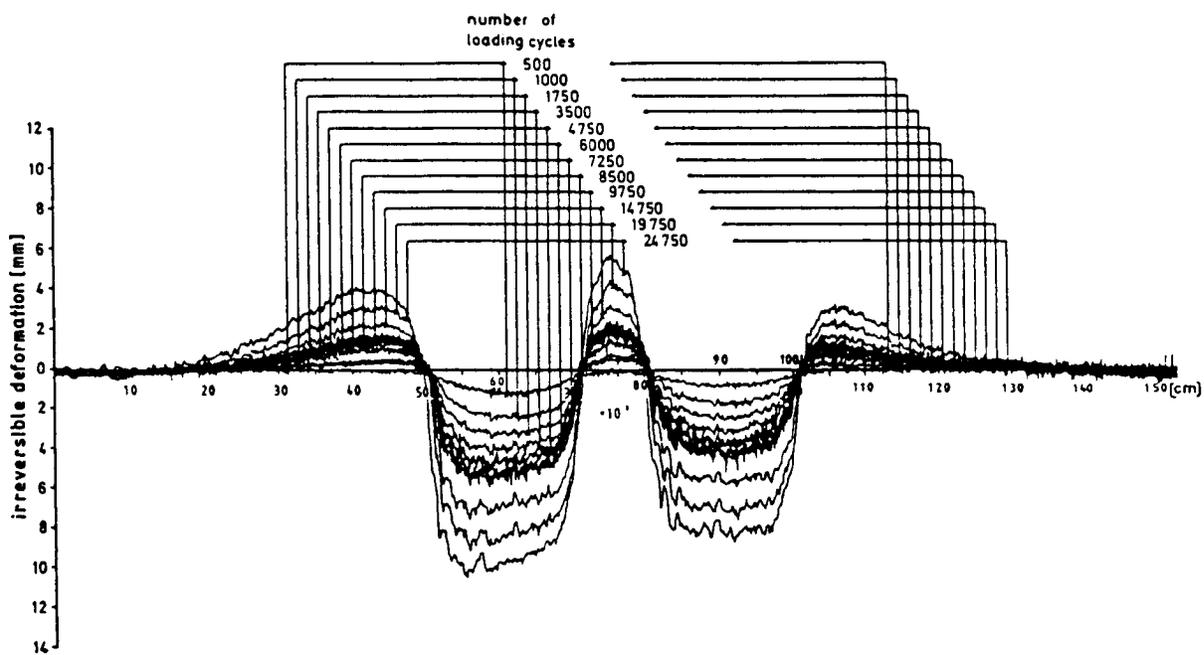
Rutting in paving materials develops gradually with increasing numbers of load applications, usually appearing as longitudinal depressions in the wheel paths accompanied by small upheavals to the sides. It is caused by a combination of densification (decrease in volume and, hence, increase in density) and shear deformation and can occur in any one or more of the pavement layers as well as in the subgrade. Trenching studies performed at the AASHO Road Test (Highway Research Board, 1962) and test-track studies reported by Hofstra and Klomp (1972) indicated that shear deformation rather than densification was the primary rutting mechanism. The importance of placing materials at high densities in order to minimize shear deformation was emphasized.

Recent work of Eisenmann and Hilmer (1987) also concluded that rutting was mainly caused by deformation flow without volume change. Figure 2.1, reproduced from the Eisenmann and Hilmer paper, illustrates the effect of the number of wheel passes on the surface profile of a wheel-track test slab<sup>2</sup>. These data enable the measurement of the average rut depth as well as the volumes of displaced material below the tires and in the upheaval zones adjacent to them. From information such as that presented in Figure 2.1, two conclusions were drawn:

- 1) In the initial stage of trafficking, the increase of irreversible deformation below the tires is distinctly greater than the increase in the upheaval zones. In this initial phase, therefore, traffic compaction has an important influence on rutting.

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<sup>2</sup>The pavement section consisted of about 23 to 24 cm of asphalt concrete (5 cm surface course and 18+ cm base course) resting on a rubber foundation of a prescribed stiffness.



**Figure 2.1 Effect of number of passes on transverse surface profile  
(After Eisenmann and Hilmer, 1987)**

- 2) After the initial stage, the volume decrement beneath the tires is approximately equal to the volume increment in the adjacent upheaval zones. This is an indication that compaction under traffic is completed for the most part and that further rutting is caused essentially by displacement with constancy of volume. This phase is considered to be representative of the deformation behavior for the greater part of the lifetime of a pavement.

Hofstra and Klomp (1972) found that the deformation through the asphalt-concrete layer was greatest near the loaded surface and gradually decreased at lower levels. Because rutting is caused by plastic flow, such a distribution of rutting with depth is reasonable: more resistance to plastic flow is encountered at greater depths and shear stresses are smaller there as well. Uge and van de Loo (1974) reported that the deformation within an asphalt layer (thickness reduction under the action of pneumatic tires) no longer increased with increasing layer thickness beyond a certain threshold (13 cm in their case). Measurements at the AASHO Road Test (Highway Research Board, 1962) indicated that the surface rut depth reached a limiting value for asphalt-concrete thicknesses of approximately 10 in. Thicker layers did not exhibit additional rutting. These results strongly suggest that, at least for reasonably stiff supporting materials, most pavement rutting is confined to the asphalt-concrete layer.

### **2.3 Factors Affecting Rutting**

Characteristics of asphalt mixtures and test or field conditions which affect rutting of asphalt-concrete pavements are summarized in Table 2.1 and discussed in following sections.

*Aggregates.* Available evidence indicates that dense aggregate gradations are desirable to mitigate the effects of rutting. When properly compacted, mixtures with dense or continuous aggregate gradations have fewer voids and more contact points

**Table 2.1. Factors affecting rutting of asphalt-concrete mixtures.**

	Factor	Change in Factor	Effect of Change in Factor on Rutting Resistance
Aggregate	Surface texture	Smooth to rough	Increase
	Gradation	Gap to continuous	Increase
	Shape	Rounded to angular	Increase
	Size	Increase in maximum size	Increase
Binder	Stiffness <sup>a</sup>	Increase	Increase
Mixture	Binder content	Increase	Decrease
	Air void content <sup>b</sup>	Increase	Decrease
	VMA	Increase	Decrease <sup>c</sup>
	Method of compaction	- <sup>d</sup>	- <sup>d</sup>
Test field conditions	Temperature	Increase	Decrease
	State of stress/strain	Increase in tire contact pressure	Decrease
	Load repetitions	Increase	Decrease
	Water	Dry to wet	Decrease if mix is water sensitive

<sup>a</sup>Refers to stiffness at temperature at which rutting propensity is being determined. Modifiers may be utilized to increase stiffness at critical temperatures, thereby reducing rutting potential.

<sup>b</sup>When air void contents are less than about 3 percent, the rutting potential of mixes increases.

<sup>c</sup>It is argued that very low VMA's (e.g., less than 10 percent) should be avoided.

<sup>d</sup>The method of compaction, either laboratory or field, may influence the structure of the system and therefore the propensity for rutting.

between particles than open<sup>3</sup> or gap-graded mixtures. For example, Brown and Pell (1974) concluded that a gap-graded mixture exhibits more deformation than a continuously graded mixture. They argued that this is due to less aggregate interlock in the gap-graded mixture. They further argued that, because aggregate interlock becomes more important at higher temperatures, gap-graded mixtures may be even more susceptible to rutting at higher temperatures, a finding apparently confirmed by test-track results.

For good rutting resistance, the surface texture of the aggregate plays an extremely important role. Particularly in thicker asphalt-bound layers and hotter climates, a rough surface texture is required. Particle shape is also important. Uge and van de Loo (1974) reported that mixtures made from angular aggregates (obtained by crushing) deformed to a minor extent and were more stable than mixtures having the same composition and grading but made from rounded aggregates (river gravel). Figure 2.2 demonstrates that, at a given void content, crushed aggregates produce stiffer mixtures. In their study, the effect of crushing on surface texture was not defined: accordingly, it is difficult to separate the effects of surface texture from those of shape.

Uge and van de Loo also used the shear creep test to investigate the stabilities of different mixtures having identical aggregate grading curves. As before, the most stable mixture was made of crushed aggregate and the least stable, of rounded aggregate. Interestingly (Figure 2.3), an intermediate composition, of which only the sand fraction was crushed, performed better than the formulation in which only the coarse aggregate was crushed, although the former contained a higher proportion of rounded components (70 percent versus 25 percent). This indicates that interparticle contact may be a more significant factor than the extent of crushing.

With increased tire pressures, axle loads, and load repetitions, there has been a resurgence of interest in the use of "large-stone" mixtures. For example, Davis (1988) has reported that some asphalt pavements constructed with soft asphalts, high volume concentrations of aggregate, low air-void contents, and large maximum aggregate size (1½ in. or larger) exhibited good rutting resistance. Based on such observations, he

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<sup>3</sup>While this appears to be the general consensus among paving engineers, there is evidence that open-graded mixtures have exhibited good rutting resistance (e.g., Hicks et al., 1983).

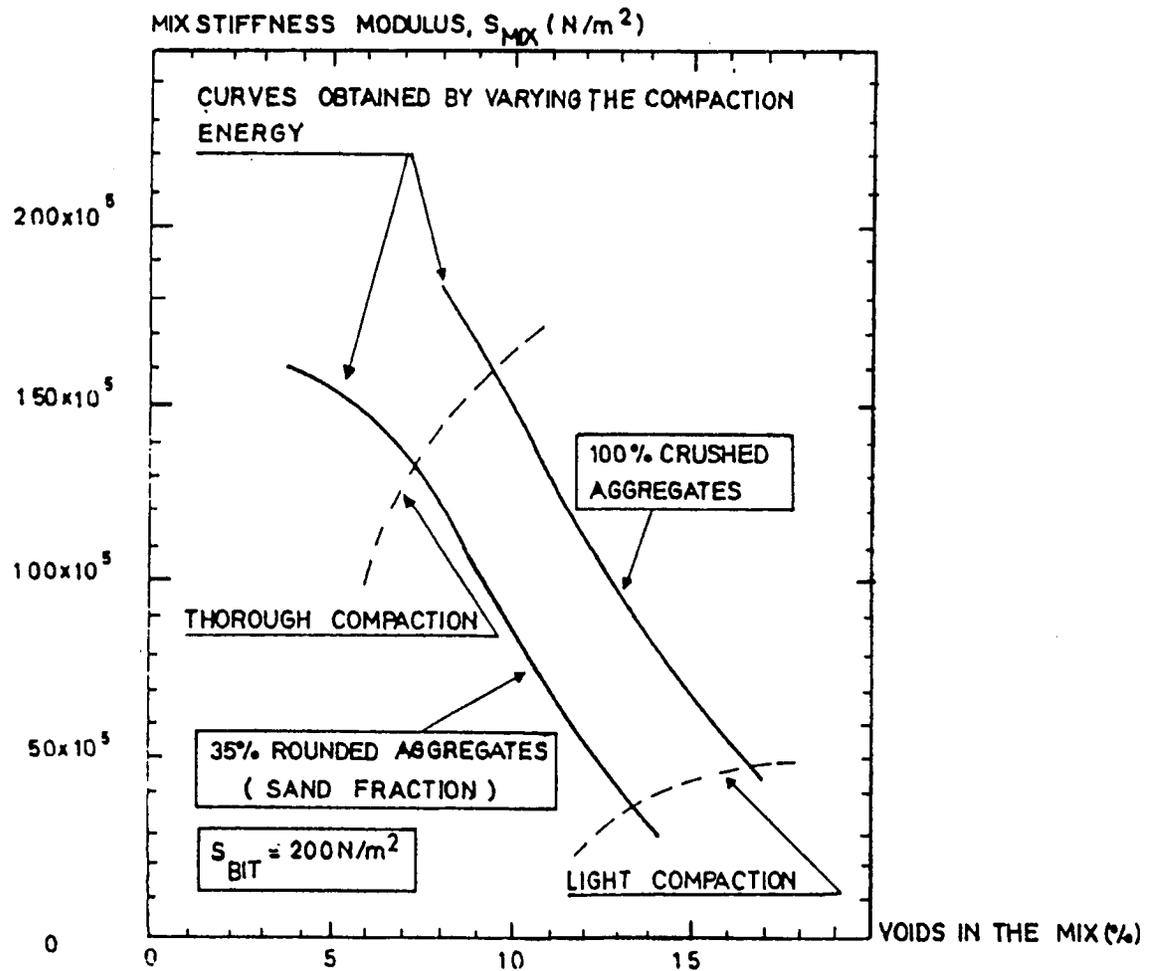


Figure 2.2 Effect of aggregate angularity and void content on mixture stiffness in compression (After Uge and van de Loo, 1974)

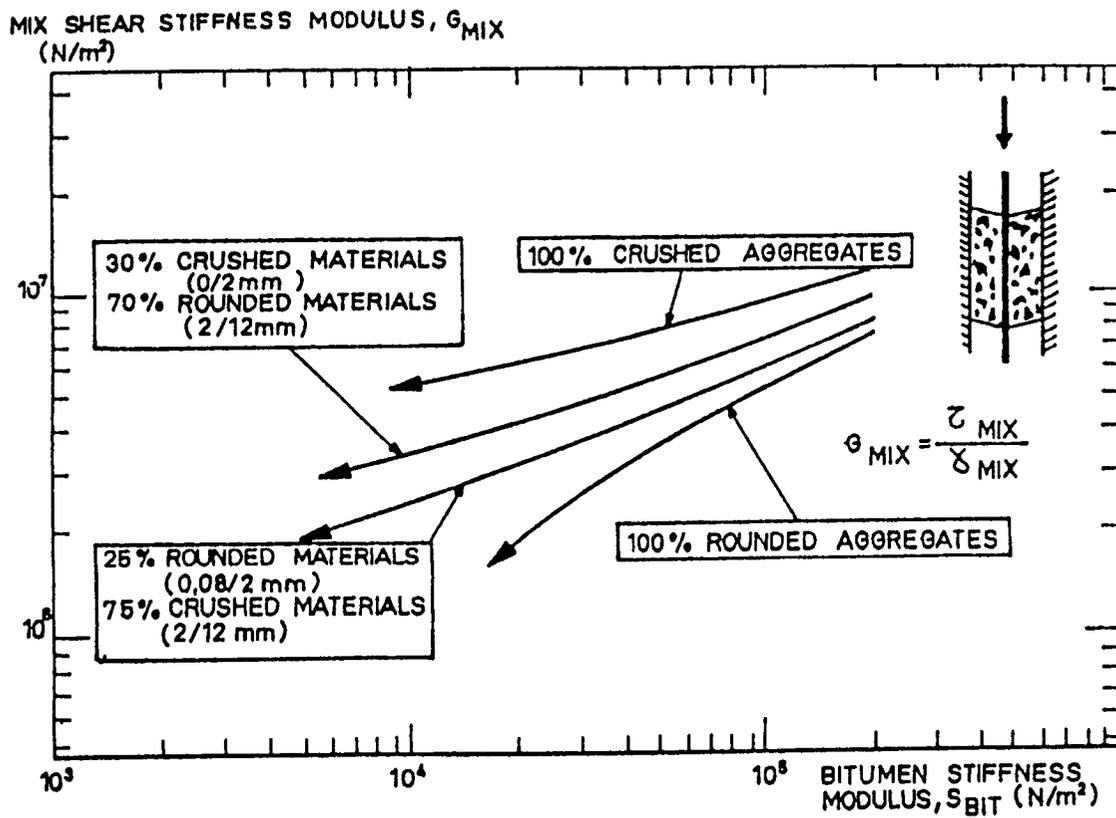


Figure 2.3 Effect of aggregate angularity and bitumen stiffness on mixture stiffness in shear (After Uge and van de Loo, 1974)

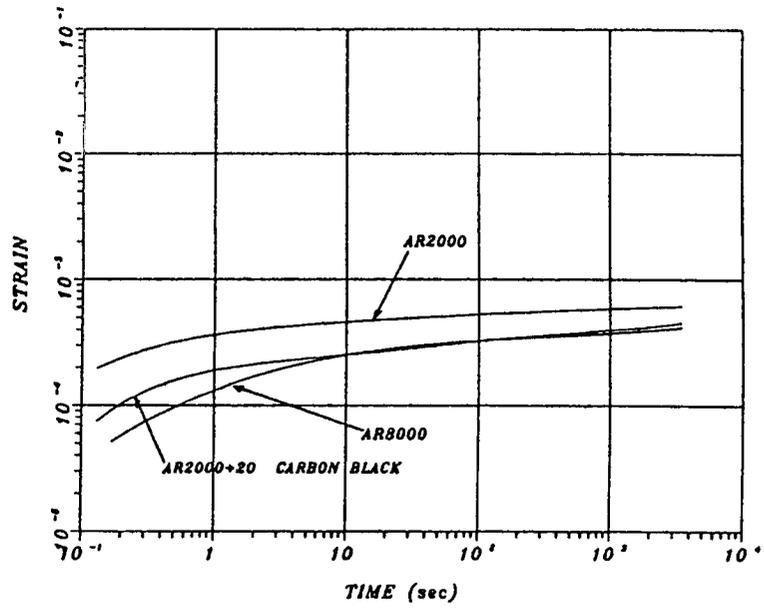
concluded that the use of larger maximum aggregate size (about two-thirds of layer thickness) would be beneficial in reducing the rutting propensity of mixtures subjected to high tire pressures.

***Binder.*** On the basis of uniaxial creep testing, Mahboub and Little (1988) concluded that less viscous asphalts make the mixture less stiff and therefore more susceptible to irrecoverable deformations, i.e., rutting. Monismith, Epps, and Finn (1985) made similar observations and recommended harder (more viscous) asphalt cements in thicker pavements and hotter climates.

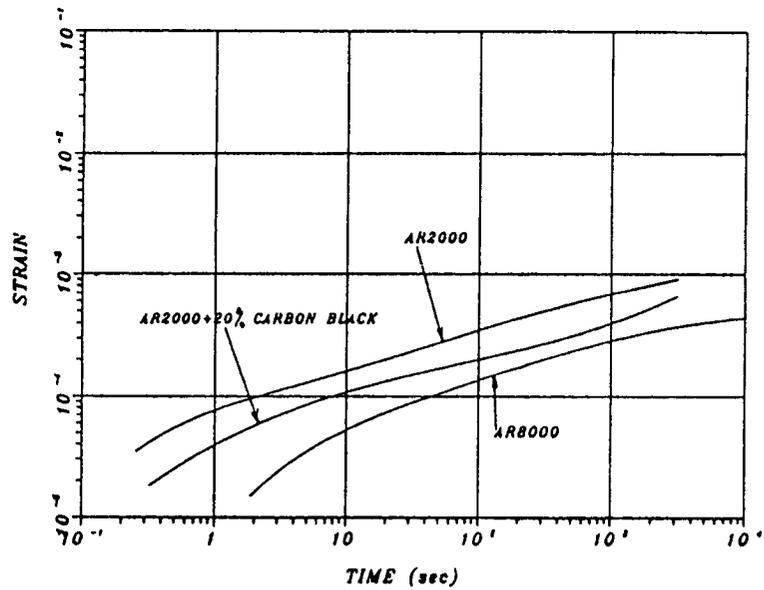
Several researchers have tried to improve rutting performance by using modifiers (polymers, microfillers, etc.) intended to increase the viscosity of the asphalt binder at high temperatures without adverse effect at low temperatures. For example, Monismith and Tayebali (1988) investigated the relative behavior of mixtures containing AR2000, AR8000, and AR2000 modified by carbon black as a microfiller. Based both on creep tests and repeated load triaxial tests, the later two mixtures afforded better resistance to permanent deformation at high temperatures than the mixture containing the AR2000 asphalt cement (Figure 2.4).

***Mixture.*** The binder content also affects the mixture's ability to resist permanent deformation. The Marshall or Hveem method is generally selected as a preliminary design tool in the determination of an adequate asphalt content. Monismith, Epps, and Finn (1985) recommended that the mixture have an asphalt content such that the air-void content would be approximately 4 percent. To preclude problems of instability and, therefore, permanent deformation, they recommended an absolute minimum of three percent air voids. These criteria must necessarily be associated with mixtures of "adequate" stability resulting from the use of high quality aggregates.

Mahboub and Little (1988) indicated that larger asphalt contents producing lower air voids increased rutting potential (Figure 2.5). They suggested that the reduction in air voids as a result of increased asphalt content indicates that void space is becoming filled with asphalt. As a result, the increase in asphalt content is equivalent to the introduction of lubricants between aggregate particles otherwise separated by a very tight network of air voids. This phenomenon causes the mixture with the higher asphalt

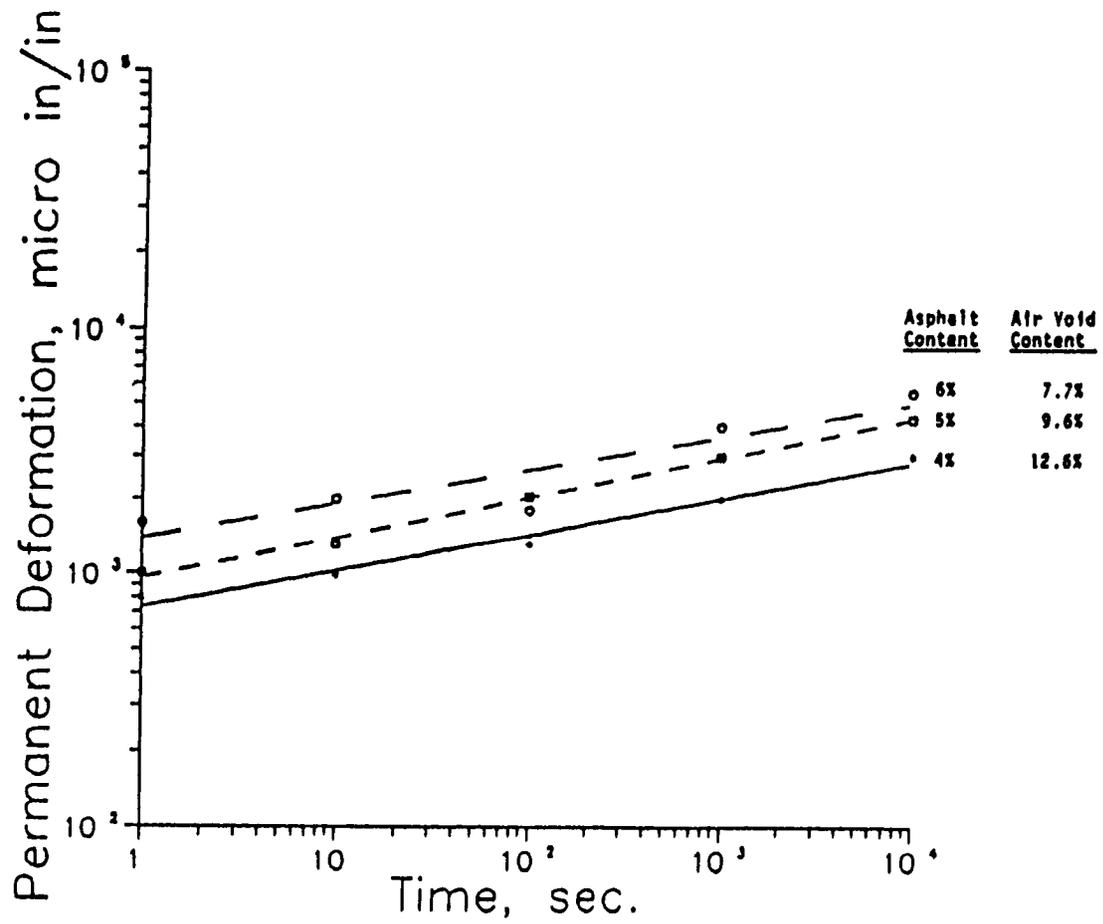


a. Creep



b. Repeated loading

Figure 2.4 Comparative response of three mixtures in creep and repeated loading at 100°F (After Monismith and Tayebali, 1988)



**Figure 2.5 Permanent-deformation trends for AC-5 and crushed limestone mixtures (After Mahboub and Little, 1988)**

content to be more susceptible to permanent deformation.

Cooper, Brown, and Pooley (1985) concluded (Figure 2.6) that good resistance to permanent deformation requires low voids in the mineral aggregate (VMA) and that the desirable grading for minimum VMA can be determined using dry aggregate tests. However, they cautioned that the lowest theoretical VMA could be undesirable as it may not allow sufficient voids in the aggregate for enough binder to ensure satisfactory compaction without the mixture becoming overfilled<sup>4</sup>.

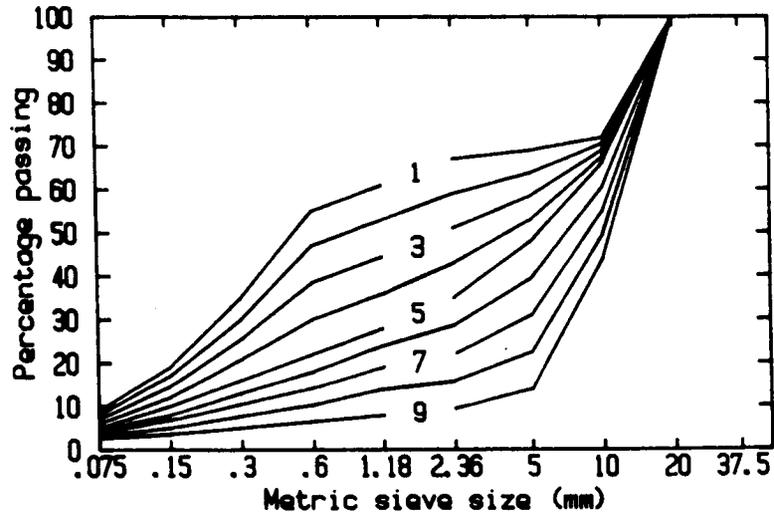
As indicated earlier, reducing air voids (up to a point) increases the resistance of the mixture to rutting. In the field, a low air-void content is generally achieved with higher compactive energy. Uge and van de Loo (1974) found that relative displacements of mineral particles occurring when an asphalt mixture is handled at high temperature (during laying or compaction) or at moderate temperature but under prolonged loading are of the same nature. Therefore, to minimize rutting propensity, they recommended the use of harsh mixtures--those of comparatively poor workability--and heavy rollers. Such a combination should result in an improved arrangement of the mineral skeleton and thereby an increase in internal friction. They concluded that harsh mixtures, thoroughly compacted after laying, will be very resistant to permanent deformation.

Linden and Van der Heide (1987) stressed the importance of proper compaction and concluded that degree of compaction is one of the main quality parameters of the placed mixture, especially for critical designs (those having a low bitumen content intended to deliver a high resistance against permanent deformation). The well-designed, well-produced mixture performs better (better durability and mechanical properties) when it is well-compacted.

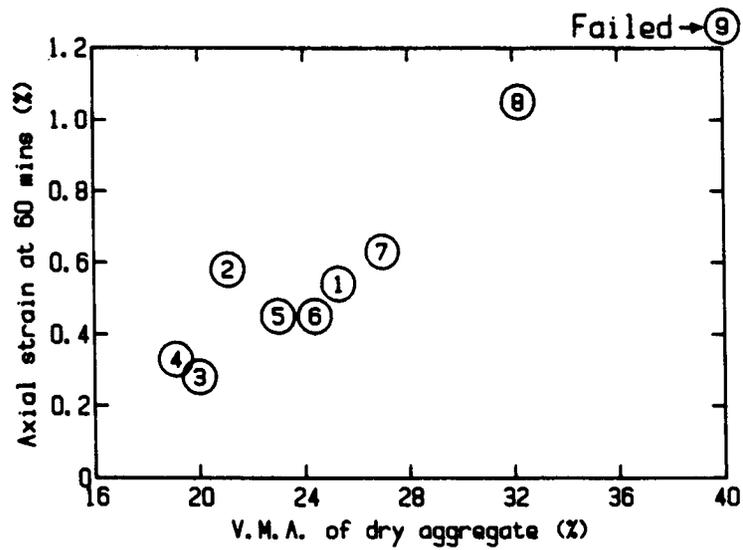
Compaction is also a critical factor in preparing specimens for laboratory evaluation. The purpose of any laboratory compaction process is to simulate, as closely as possible, actual compaction produced in the field. Factors such as the orientation and interlocking of aggregate particles, the extent of interparticle contact, air-void content and void structure, and number of interconnected voids should be closely reproduced.

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<sup>4</sup>The Asphalt Institute (1984) provides guidelines for minimum VMAs, depending on the maximum aggregate size.



a. Grading curves



b. VMA of dry aggregate

Figure 2.6 Effect of voids in the mineral aggregate on resistance to deformation for nine transitional mixtures (After Cooper, Brown, and Pooley, 1985)

Several studies have sought to determine the extent to which various types of laboratory compaction simulate field conditions. The most recent is the AAMAS study (Von Quintus et al., 1988) which compared field cores with laboratory specimens compacted using the Texas gyratory-shear compactor, the California kneading compactor, the mobile steel wheel simulator, the Arizona vibratory/kneading compactor, and the Marshall hammer. The investigators ranked compaction devices, based on their abilities to consistently simulate the engineering properties of field cores, as follows:

- 1) Texas gyratory-shear compactor,
- 2) California kneading compactor and mobile steel wheel simulator,
- 3) Arizona vibratory/kneading compactor, and
- 4) Marshall hammer.

The AAMAS evaluation was based primarily on the results of creep tests conducted in the diametral mode. Because complexity of the stress state makes diametral creep test results difficult to properly interpret (Sousa, 1990), other testing would have provided valuable confirmation of the AAMAS findings. Additional testing of field cores after in-service conditioning by traffic and weathering<sup>5</sup> would also have been helpful.

Vallerga and Zube (1953) evaluated the effect of laboratory compaction method using the Hveem stabilometer to measure mixture stability. They concluded that kneading compaction was more effective in obtaining high densities and stabilities than the other two methods (double plunger and impact). Furthermore, the asphalt content at maximum stability was less with kneading compaction than with other compaction methods in use at the time (1953). Field densities from in-service pavements, under traffic for several years, were invariably larger than laboratory densities of freshly prepared mixtures. Accordingly, the investigators emphasized that the laboratory compaction method and compaction effort must produce specimens representative of pavements after conditioning by traffic loading. Finally, they concluded that kneading compaction produced specimens with essentially the same characteristics as traffic compacted specimens.

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<sup>5</sup>In the AAMAS study, field cores were extracted very shortly (one day) after construction.

Monismith and Tayebali (1988) compared shear creep moduli of specimens cored from in-service pavements with those of specimens compacted in the laboratory with the California kneading compactor. The laboratory specimens were fabricated from materials that had been mixed in the field. Both types of specimens were tested in an uniaxial simple shear device. The authors concluded that the shear response of field cores was essentially the same as that of laboratory specimens compacted by kneading compaction.

These studies demonstrate that, to prepare representative laboratory specimens for permanent-deformation testing, "shearing" deformations are essential to the compaction process. Moreover, they demonstrate the importance of measuring permanent-deformation characteristics under conditions representative of those occurring in the field. Hence the densification expected in the mixture due to repeated trafficking must be properly duplicated in the laboratory.

*Test/Field Conditions.* Temperature has been found to have a significant effect on rutting. Hofstra and Klomp (1972) determined from test-track measurements that rutting increased by a factor of 250 to 350 with a temperature increase from 68°F to 140°F (20°C to 60°C). Linden and Van der Heide (1987) reported a significant increase in rutting in Europe during the very hot summers of 1975 and 1976.

Researchers have recognized the need to conduct laboratory tests at temperatures within the high-temperature range of those encountered in the field. Bonnot (1986) selected a test temperature 60°C for wearing-course asphalt concrete and 50°C for base courses. These temperatures were chosen to be relatively high to reproduce the most unfavorable conditions expected in France.

Similarly, Mahboub and Little (1988) conservatively selected the hottest pavement profile to represent critical conditions. Other assumptions about the accumulation of permanent deformation in Texas pavements included the following:

- 1) Permanent deformation occurs daily over the time interval from 7:30 a.m. to 5:30 p.m.;

- 2) Permanent deformation occurs only in the period from April to October, inclusive; and
- 3) Permanent deformation can be ignored at temperatures below 50°F.

All the previously discussed factors affect mixture resistance to permanent deformation, and all must be properly considered in order to reduce the rutting propensity of asphalt-aggregate mixtures. At the same time, it must be emphasized that the states of stress and strain caused by traffic loading also significantly influence pavement rutting.

Eisenmann and Hilmer (1987) suggested that rutting can be controlled in two ways: 1) by optimization of the asphalt mixture and the pavement construction, and 2) by optimization of the design of heavy duty trucks. They also suggested that wheel load and tire inflation pressure have a strong influence on rutting. From wheel-track results, they developed the following relationship:

$$y = a + b(N)^{as} \quad (2.1)$$

where  $y$  is the rut depth,  $N$  is the number of load repetitions, and  $a$  and  $b$  are laboratory-determined parameters.

Figure 2.7 illustrates the influence of average contact pressure (obtained from different loads and different tire pressures) on the parameter,  $b$ . For example, increasing the contact pressure of a twin tire from 0.6 MPa (87 psi) to 0.9 Mpa (130 psi) increases the parameter,  $b$ , by a factor of about 3, indicating a significant increase in rut depth.

Normally a mixture is designed for a given intensity and distribution of traffic based on past correlations with field performance. Changes in the distribution of traffic, especially increases in the proportion of heavy trucks, may increase the rate of rutting even if the pavement was properly designed and constructed originally. Such an example is demonstrated in an investigation of the causes of early rutting in a pavement in Dubai (Vallerga et al., 1989). After extensive laboratory and field studies, it was

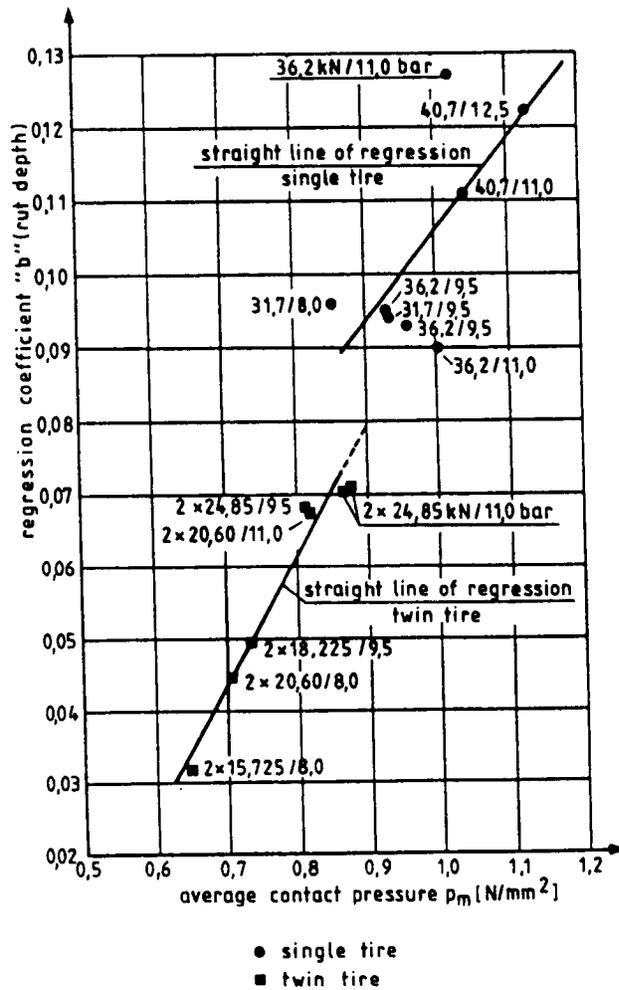


Figure 2.7 Effect of average tire contact pressure on rutting rate (regression coefficient "b") (After Eisenmann and Hilmer, 1987)

determined that the mixture and pavement had originally been well designed according to existing methodologies. The accelerated rutting was attributed to heavier-than-expected loads (up to 80,000-pound axle loads as compared to the 20,000-pound axle loads used for design) and higher tire pressures, loading conditions beyond those for which the available methodology was applicable. This study indicated that the instability could have been anticipated had test specimens been compacted in the laboratory to the actual densities obtained in the pavement after 10 months of trafficking.

## **2.4 Permanent-Deformation Criteria**

The Federal Highway Administration (1979) has classified rutting into four levels of severity: 1) hydroplaning (0.2 to 0.25 in.); 2) low (0.25 to 0.5 in.); 3) medium (0.5 to 1.0 in.); and 4) high (1.0 in.) Many researchers, however, consider that the only reasonable standard is that associated with hydroplaning. For pavements with crown slopes of the order of 2 percent and rut depths of about 0.5 in., ponding is sufficient to cause automobiles travelling at speeds of 50 mph or more to hydroplane (Barksdale, 1972).

Allowable rut depth may be limited by both safety and structural considerations. For example, pavement failure in the United Kingdom is defined as a rut depth of 0.75 to 0.8 in. (19-20 mm) measured by a 6-ft. (1.8 m) straightedge (Lister and Addis, 1976). It is suggested that rut depths up to approximately 0.4 in. (10 mm) do not cause significant loss of structural strength. For a cross slope of 2.5 percent (generally used in the United Kingdom), Lister and Addis (1976) have found that ruts deeper than approximately 0.5 in. (13 mm) result in ponding of water which could cause hydroplaning or loss of skid resistance. This corroborates Barksdale's finding as noted above. Verstraeten et al. (1977) have determined that the rut slope (ratio of rut depth to one-half its width) should not exceed 2 percent for good riding quality.

In order to limit rutting in asphalt-bound layers to acceptable levels, some investigators have suggested limiting values of mixture stiffness as measured in the laboratory creep test at a specific time of loading and a specific temperature. Examples of such criteria are:

Reference	Temperature, °C	Time, min.	Applied Stress, $\sigma_0$ , Mpa (psi)	Mix Stiffness Mpa (psi)
Viljoen et al. (1981)	40	100	0.2 (30)	$\geq 80$ (12,000)
Kronfuss et al. (1984)	40	60	0.1 (15)	$\geq 50-65$ (7,500-10,000)
Finn et al. (1983)	40	60	0.2 (30)	$\geq 135$ (20,000)

Criteria of this type must be associated with specific traffic and environmental conditions and should not be adopted without careful evaluation.

### 3

## Rutting Prediction

With increases in axle loads, load repetitions, tire pressures, and asphalt-concrete thicknesses, a need has developed for methodology to predict rut depths in advance of construction to mitigate potential safety problems, e.g., hydroplaning. Concomitant with the development of analysis procedures which permit estimates of stresses, strains, and deformations resulting from traffic loads, pavement design systems have evolved which include provisions for rutting considerations. These have been referred to as analytically-based, mechanistic, or mechanistic-empirical procedures. A number of these procedures include criteria for limiting values of subgrade strain to levels that preclude rutting at the pavement surface. Examples include the Shell procedure (Claessen et al., 1977); the Asphalt Institute procedure (Shook et al., 1982); and the State of Kentucky methodology (Southgate et al., 1977). Some have recommended limitations on vertical subgrade stress (rather than strain), e.g., Barksdale and Miller (1977). Others have utilized statistically-based rut depth prediction equations. For example, Saraf et al. (1976) presented such an equation that incorporated surface deflection as computed from elastic layered analysis.

Design limitations on strain or stress are based on the assumption that, if the maximum vertical compressive strain or stress at the surface of the subgrade is less than a critical value, then rutting will be limited to a tolerable level for a specified number of load applications. Unfortunately, such methodology does not necessarily preclude rutting which might occur in the asphalt-bound layer. The Shell method (Claessen et al., 1977) exemplifies a procedure which attempts to improve the above process by including additional analysis to estimate the amount of rutting occurring in the asphalt-bound layer. This predictive methodology is an example of one of two analytical procedures which have evolved to estimate the amount of rutting. The procedure makes use of layered system elastic analysis and represents one of a number of such procedures termed *layer-strain*

predictive methodologies. The second approach makes use of closed form *viscoelastic* analyses. Both are described in following sections.

### 3.1 Layer-Strain Procedure

The layer-strain method consists of predicting rut depths using permanent-deformation characteristics determined from laboratory tests together with an analysis procedure for the pavement structure using either linear or nonlinear elastic theory. The general principle of this method was first proposed by Barksdale (1972) and Romain (1972). While nonlinear elastic theory should provide more accurate results (Brown and Bell, 1977), it has not been used very extensively because of its added complexity.

To predict the amount of permanent deformation that would occur after a given number of wheel load applications, each layer of the pavement structure is divided into several sublayers, and the stress state is calculated at the center of each sublayer directly beneath the wheel load (Figure 3.1) using elastic analysis. With the average stress state at the center of each sublayer, the corresponding axial plastic strain can be ascertained from the results of laboratory tests. The total rut depth for a given number of load repetitions is obtained by summing the products of the average plastic strain occurring at the center of each sublayer and the corresponding sublayer thickness, i.e.,:

$$\Delta p = \sum_{i=1}^n [(\epsilon_i^p)(\Delta z_i)] \quad (3.1)$$

where  $\Delta p$  is the total rut depth,  $\epsilon_i^p$  is the average plastic strain in the  $i$ th sublayer,  $\Delta z_i$  is the thickness of the  $i$ th sublayer, and  $n$  is the total number of sublayers. This approach has been adopted in various forms by many researchers and, as noted above, the Shell design methodology represents a practical example of the use of this procedure.

The layer-strain method is considered a simplified engineering approach for predicting rut depth which permits the flexibility of using either linear or nonlinear elastic analysis. Further improvements are necessary to extend the prediction of rut depth from the centerline of the loading to the entire rutting zone. Currently the laboratory test procedure makes use of some form of axial compression, either creep or repeated loading. If predictions are to be extended beyond the center of the loaded area, the laboratory procedure must incorporate provision for states of stress which include significant shear components: such stress states exist off centerline and are particularly important near the tire edges. It also implies that new functions relating stress to plastic deformation will be

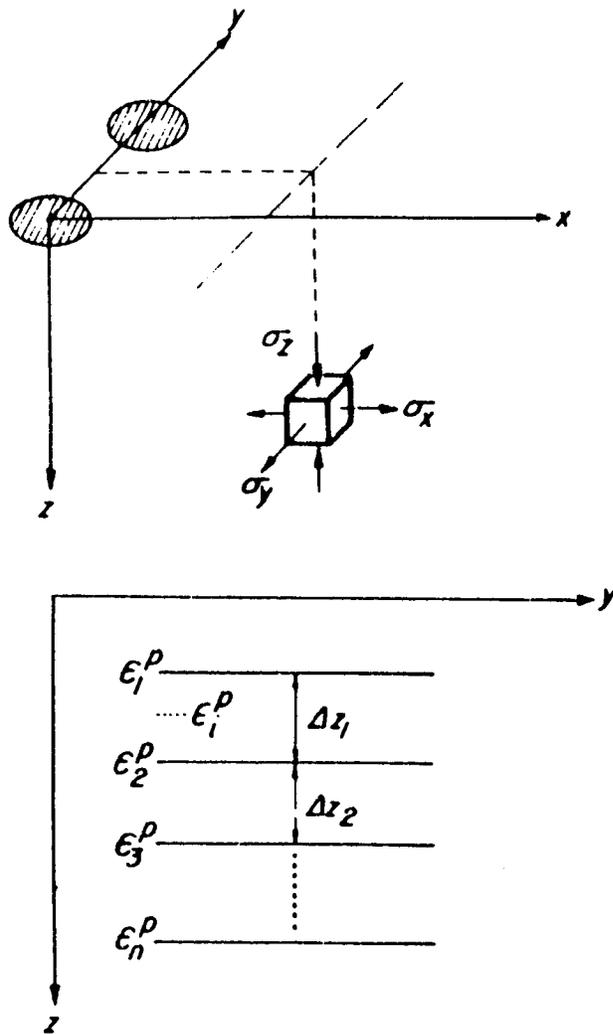


Figure 3.1 Diagram of pavement system used to estimate permanent deformation

required to take shear stresses into consideration.

### **3.2 Viscoelastic Methodologies**

In this approach moving wheel loads can be considered in conjunction with time-dependent material properties to define the states of stress and strain at particular points in the pavement structure. Material properties can be defined either in terms of models consisting of finite numbers of Maxwell and/or Kelvin elements in various arrangements or in terms of generalized compliance relationships. An important advantage of this approach is that moving wheel loads can be considered directly. This results in the correct time-rate of loading to be applied to each material element and permits estimates to be made of the lateral plastic flow of material from beneath the moving wheel.

While nonlinear viscoelastic response characteristics may provide a more realistic estimate of pavement response, the associated mathematical complexities have limited past analyses to linear characterizations (Barksdale and Leonards, 1967; and Elliot and Moavenzadeh, 1971). In this approach, material properties within a given layer are assumed to be the same throughout the layer regardless of whether the material is in tension or compression. Another example of this methodology is embodied in the VESYS procedure (Kenis, 1977).

If viscoelastic analysis is used, generalized linear viscoelastic response is tractable, and several authors have used or proposed this method (e.g., Moavenzadeh and Elliot, 1972). The calculations tend to be time consuming, however, and the assumption of linearity itself is questionable (Thrower, 1977). A nonlinear viscoelastic model seems even more prohibitive, in terms of both computational effort and the scale of laboratory work necessary to establish appropriate nonlinear, time-dependent constitutive equations. A viscoplastic theory, coupled with the use of finite-element methods (Thrower, 1977) seems to have the same drawbacks. More recently, Thrower et al. (1986) and Nunn (1986) have suggested that basing estimates of the accumulation of permanent deformation on viscous properties of the asphalt concrete has the potential to provide reasonable estimates of rutting.

### **3.3 Comparison of Layer-Strain and Viscoelastic Methodologies**

Common to both layer-strain and viscoelastic methodologies are 1) the determination of the states of stress in the pavement structure and 2) the specification of permanent-deformation characteristics of the materials in the pavement as a function of stress state, load repetitions, temperatures, and so forth. Table 3.1 summarizes some of the models for

determining pavement response together with the permanent-deformation equations (characterizing the permanent-deformation behavior of asphalt-concrete mixtures) proposed by their authors.

The layer-strain method has been considered to be a "reasonable" approach for predicting rut depth, at least for comparative purposes: it provides the added flexibility of allowing use of either linear or nonlinear elastic theory. Although the viscoelastic method is theoretically more appealing, its complexity and the relatively poor agreement between measured and predicted values thus far demonstrated indicate that it does not present a significant advantage over the layer-strain method. However, if permanent-deformation equations (generalized permanent deformation laws) are developed based on tests that apply states of stress comparable to those encountered in pavements near the tire edges and if viscoelastic models are developed that can incorporate these laws, more accurate predictions are expected.

### 3.4 Calculation of Pavement Response

Determining the states of stress and strain in a pavement section under a load is an essential step in any rut-depth predictive methodology. Use of a suitable mathematical model of the pavement system and realistic material properties are closely interrelated: methods for predicting rutting must be developed with this in mind.

Difficulties in duplicating the appropriate stress states have been carefully considered by Brown and Bell (1977). They have pointed out that Barksdale (1972) and Romain (1972) related permanent strain to vertical and horizontal stresses. They suggested, furthermore, the use of stress invariants as the most appropriate method of representing the correct stress state for materials characterization. The use of stress invariants is particularly advantageous in the tension zone in the bottom of bituminous layers and also for predicting rutting away from the axis of symmetry of loading. Following this approach, the stress conditions at any point can be characterized by the mean normal stress,  $p$ , and the octahedral shear stress,  $\tau_{oct}$ , where:

$$p = 1/3(\sigma_1 + \sigma_2 + \sigma_3) \quad (3.2a)$$

$$\tau_{oct} = (1/3) * [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2} \quad (3.2b)$$

where  $\sigma_1$ ,  $\sigma_2$ , and  $\sigma_3$  are the principal stresses existing at the point. For simplicity, a shear stress term,  $q$ , can be defined as:

$$q = \left(\frac{3}{\sqrt{2}}\right) * (\tau_{oct}) \quad (3.3)$$

Table 3.1. Summarized overview of the models and permanent deformation equations used by several authors.

Author	Pavement Analysis	Permanent Deformation Equation	Variables	Laboratory Test	Observations
Kirwan Snaith Glynn (1977)	DEFPV Nonlinear finite element Layer strain method	$\epsilon_p = A \Delta^b$	$\epsilon_p$ = induced axial permanent strain after an elapsed time N A = function of elapsed time and material constant for the material b = applied axial compressive stress	Uniaxial compression dynamic loading creep test	Calculated values of rut depth higher than obtained in Nottingham Test Track
Monismith Inkabi Freeme McLean (1977)	ELSYM Layered elastic theory Layer strain method	$\epsilon_p^2 = [\delta(T) N^a \bar{\sigma}^{n-1}] \cdot [\sigma_z - \frac{1}{2}(\sigma_x + \sigma_y)]$	$\epsilon_p^2$ = vertical permanent deformation $\delta(T)$ = function of temperature coefficient determined experimentally $\alpha$ = number of stress repetitions N = equivalent stress defined as a function of $\sigma_1, \sigma_2$ and $\sigma_3$ t = loading time	Repeated load triaxial compression test	Consideration of traffic axle load and lateral distribution
Brown Bell (1979)	Computer program DEFPV Nonlinear finite element Layer strain method	$\epsilon_p = (\frac{q}{a})^b (N)$	$\epsilon_p$ = permanent shear strain q = deviator stress a,b = constant N = number of load applications	Axial repeated load tests	Relatively good agreement in test track

Table 3.1. Summarized overview of the models and permanent deformation equations used by several authors, continued.

Meyer Haas (1977)	Computer program FEPAVE II Finite element Layer strain method	$\epsilon_p = F(\sigma_1, \sigma_3, T, AV, N) \pm E$	$\epsilon_p$ = axial permanent deformation $\sigma_1$ = vertical stress $\sigma_3$ = lateral stress $T$ = temperature $AV$ = air voids $N$ = number of load applications $E$ = error of estimate	Repeated load triaxial test	Measured values for rut depth on the Brampton Test Road sections. Good agreement between measured and predicted.
van de Loo (1976)	BISAR Elastic layer theory	$\epsilon_p = c \sigma N^a$	$\epsilon_p$ = axial permanent deformation $c$ = constant $\sigma$ = axial stress level (15 PSI) $N$ = number of load applications $a$ = constant	Axial creep test	Basis of SHELL method. Generally overestimates rut depth.
Kenis (1977)	VESYS Probabilistic linear viscoelastic solution	$\epsilon_p(N) = e \mu N^{-\alpha}$	$\epsilon_p(N)$ = permanent strain per pulse $\alpha$ = 1 - S $S$ = slope of the line on a log-log plot of permanent strain versus N $e$ = peak haversine load strain for a load pulse of duration d = 0.1 sec $\mu$ = IS/e $I$ = intercept	Uniaxial repeated load tests	Basis of the VESYS approach

Table 3.1. Summarized overview of the models and permanent deformation equations used by several authors, continued.

<p>Fræncken (1977)</p>	$e_p(t) = At^B + C(\exp Dt - 1)$ <p>(high stresses)</p> $e_p(t) = At^B$ <p>(low stresses)</p>	<p><math>e_p(t)</math> A,B,C,D</p> <p>A</p> <p>B</p> <p><math>\sigma_{VM}</math></p> <p><math>\sigma_{VL}</math></p>	<p>permanent strain parameters</p> <p><math>115 (\sigma_1 - \sigma_3) /  E^* </math></p> <p><math>.182 + .294 (\sigma_{VM} - \sigma_{VL})</math></p> <p>maximum stress plastic failure threshold</p>	<p>Triaxial dynamic tests</p>	<p>Method used to determine rutting propensity in mixes</p>
<p>Verstraeten Romain Veverka (1982)</p>	$e_p(t) = A \left( \frac{t}{10^3} \right)^B = \frac{C(\sigma_1 - \sigma_3)}{ E^*  * \left( \frac{t}{10^3} \right)^B}$	<p><math>e_p(t)</math></p> <p>A</p> <p>B</p> <p>C</p> <p><math> E^* </math></p> <p><math>\sigma_1</math></p> <p><math>\sigma_3</math></p>	<p>permanent strain at time t (in sec) is a coefficient depending on the mix composition and on the experimental conditions (stresses, frequency, temperature); it characterizes the susceptibility of the mix to rutting</p> <p>is a coefficient varying between .14 and .37</p> <p><math>f[V_b / (V_b + V_v)]</math></p> <p>modulus of the mix</p> <p>amplitude of vertical stress</p> <p>lateral stress</p>	<p>Triaxial dynamic tests</p>	<p>Acceptable correlation with rut depth measure in 16 in-service roads</p>

Table 3.1. Summarized overview of the models and permanent deformation equations used by several authors, continued.

Huschek (1977)	BISAR Elastic layer theory Layer strain theory	$e_{irr} = c \cdot \sigma t^A$ $e_{irr}(T, \Delta t_1, t) = \frac{\sigma \Delta t_1}{[\eta(T, t)] t^{1-A}}$ $\eta(T, t) = \frac{c \cdot A}{(c \cdot A)}$	$e_{irr}$ $c$ $A$ $\sigma$ $\eta$ $T$ $\Delta t_1$	permanent deformation constant consolidation characteristic stress level viscosity temperature time of loading	Uniaxial creep tests Cyclic load creep test	Asphalt mix is represented by a Maxwell element: spring and dashpot in series
Thrower (1977)	Viscoelastic theory Separative method	$\dot{e}_{ij} = \frac{\sigma_{ij}}{2\eta} \quad i \neq j$ $\dot{e}_{ij} = \frac{\sigma_m}{3\chi} + \frac{(9\sigma_{ij} - \sigma_m)}{18\eta}$	$\dot{e}_{ij}$ $\sigma_{ij}$ $\sigma_m$ $\chi$ $\eta$	rate of deformation state of stress isotropic mean stress coefficient of "volume viscosity" coefficient of shear velocity		
Battiato et al. (1977)	Computer program MOREL Viscoelastic theory Two layer viscoelastic incompressible system	$J(t) = J_1 t^\alpha$ $u_{ijk}^{perm} = \frac{1}{\eta_r} \varepsilon_{ik}(y, z)$	$J(t)$ $t$ $J_1$ $\alpha$ $u_{ijk}^{perm}$ $\eta_r$ $\varepsilon_{ik}(y, z)$	creep compliance function time shear creep parameters slope of line on a log-log between $J(t)$ and time permanent deformation shear viscosity of the Maxwell element in series tensor function	Uniaxial creep tests	Asphalt mix is represented by a Maxwell model

Table 3.1. Summarized overview of the models and permanent deformation equations used by several authors, continued.

Mahboub, Little (1988)		$\frac{\epsilon_{sp}}{N} = a\sigma^b$		$\epsilon_{sp}/N$ = = accumulated, viscoplastic deformation per cycle $\sigma$ = peak cyclic stress $a, b$ = regression parameters	Uniaxial creep tests	
Tseng, Lytton (1986)		$\epsilon_e = \epsilon_0 \exp \left[ - \left( \frac{\rho}{N} \right)^p \right]$		$\epsilon_e$ = permanent strain $N$ = load cycles $\epsilon_0, \rho, \beta$ = regression parameters	Repeated load testing	
Lai, Anderson (1973)		$\epsilon_{sp} = a(\sigma)t^b$		$\epsilon_{sp}$ = viscoplastic strain $t$ = time $a(\sigma)$ = $b_1\sigma + b_2\sigma^2$ $\sigma$ = creep stress $b_1, b_2$ = regression constants	Uniaxial creep tests	
Célad (1977)	ERDT/ESSO Three layer elastic system	$\ln \dot{\epsilon} = A + B \cdot \ln \sigma_{vm} + C \cdot \sigma_H + D \cdot T$		$\dot{\epsilon}$ = rate of permanent deformation $\sigma_{vm}$ = compressive vertical stress $\sigma_H$ = compressive horizontal stress $A, B, C, D$ = coefficients $T$ = temperature	Dynamic creep tests	Developed isocrep curves

Table 3.1. Summarized overview of the models and permanent deformation equations used by several authors, continued.

Khedr (1986)	OSU model	$\frac{\epsilon_p}{N} = A_d N^{-m}$	$\epsilon_p$ $N$ $A_d$ $m$	permanent strain number of load cycles material properties function of resilient modulus and applied stress material parameter	Multi-step dynamic tests	
Uzan (1982)		$\epsilon_p(N) = \epsilon_r \mu N^{-\alpha}$	$\epsilon_p(N)$ $\epsilon_r$ $N$ $\alpha, \mu$	permanent strain for Nth repetition resilient strain number of repetitions characteristics of materials based on intercept and slope coefficients	Repeated load testing	
Leahy (1989)		Statistically derived predictive models for permanent strain, $\epsilon_p$ $\epsilon_p = f(T, \sigma_d, V_{air}, N, \eta_{asp}, P/W_{asp})$	$\epsilon_p$ $\epsilon_r$ $a, b$ $\alpha, \mu$ $T$ $\sigma_d$ $V_{air}$ $\eta_{asp}$ $P/W_{asp}$	plastic strain resilient strain experimentally determined coefficients relating $\epsilon_p$ to load repetitions, N (VESYS) coef. mix variables, e.g., asphalt content degree of compaction temperature deviator stress volume of air asphalt viscosity effective asphalt content	Repeated load and creep, axial testing	Determined effect of mix variables on both $\epsilon_p$ and $\epsilon_r$

In the triaxial compression test, the shear stress,  $q$ , is equal to the deviator stress,  $\sigma_1 - \sigma_3$ . The mean normal stress,  $p$ , is associated with volume change whereas  $q$  is associated with shear distortion. Similarly, the strain invariants corresponding to  $p$  and  $q$  are volumetric strain ( $v$ ) and shear strain ( $e$ ), defined as:

$$v = \epsilon_1 + \epsilon_2 + \epsilon_3 \quad (3.4)$$

$$e = \left(\frac{2}{3}\right) * [(\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2 + (\epsilon_3 - \epsilon_1)^2]^{1/2} \quad (3.5)$$

where  $\epsilon_1$ ,  $\epsilon_2$ , and  $\epsilon_3$  are the principal strains at the point. In situ permanent strains develop as a result of the combination of volume change and shear distortion.

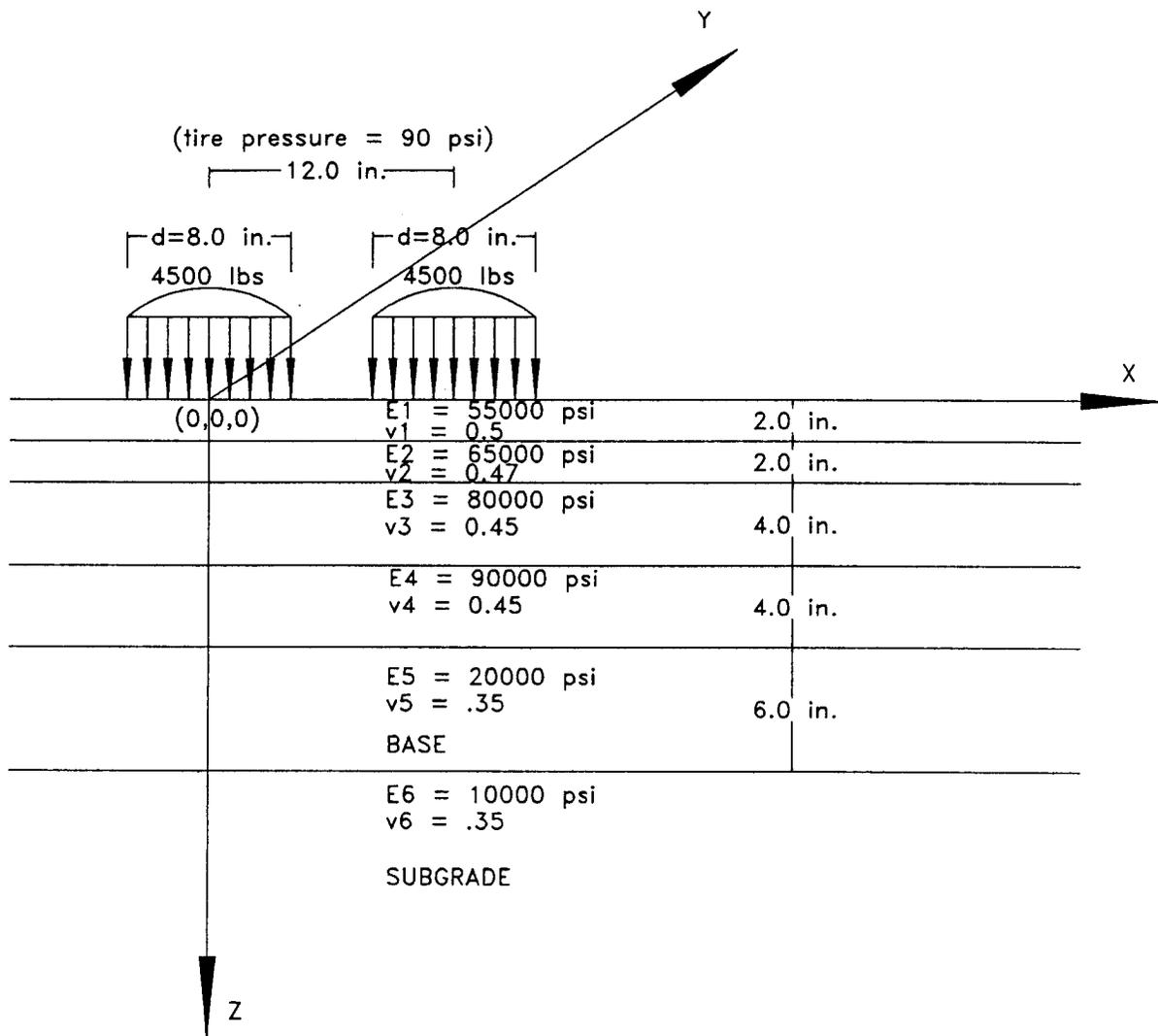
Variations of  $p$ ,  $q$ ,  $e$ , and  $v$  with depth and away from the centerline of a dual tire assembly can be easily illustrated using multilayer elastic analysis. The material properties and pavement cross-section for one such example are shown in Figure 3.2 and Table 3.2, respectively. Using the ELSYM multilayer elastic program (Ahlborn, 1972), loads were applied on dual tires with a contact stress of 90 psi. Three-dimensional plots illustrating the variation of  $p$ ,  $q$ ,  $e$ , and  $v$  in the upper part of the pavement section are shown in Figures 3.3 through 3.6.<sup>1</sup>

It is evident that the shear components,  $q$  and  $e$ , exhibit maximum values close to the surface and near the edges of the tires, indicating a strong tendency for shear distortion. Furthermore, near the surface, the mean normal stress,  $p$ , is small away from the centerline of the tire and the volumetric strain,  $v$ , is almost negligible, indicating little tendency for volume change. Volume change is associated with poor compaction, and shear strain is associated with high shear stresses in the pavement. The advantage of using  $p$  and  $q$  stress invariants is that tensile and off-axis principal stresses cannot always be directly reproduced in the triaxial test. However, some of the corresponding values of  $p$  and  $q$  can be, and  $p$  and  $q$  in the pavement structure can be calculated using elastic layered theory or finite element programs.

Brown and Bell (1977) found that substantial errors in both  $p$  and  $q$  (and, hence, plastic strain) develop if shear stresses are ignored in the determination of the state of stress, underestimating the plastic strain by as much as 40 percent. This emphasizes the importance of testing materials over the entire range of stresses expected in the field.

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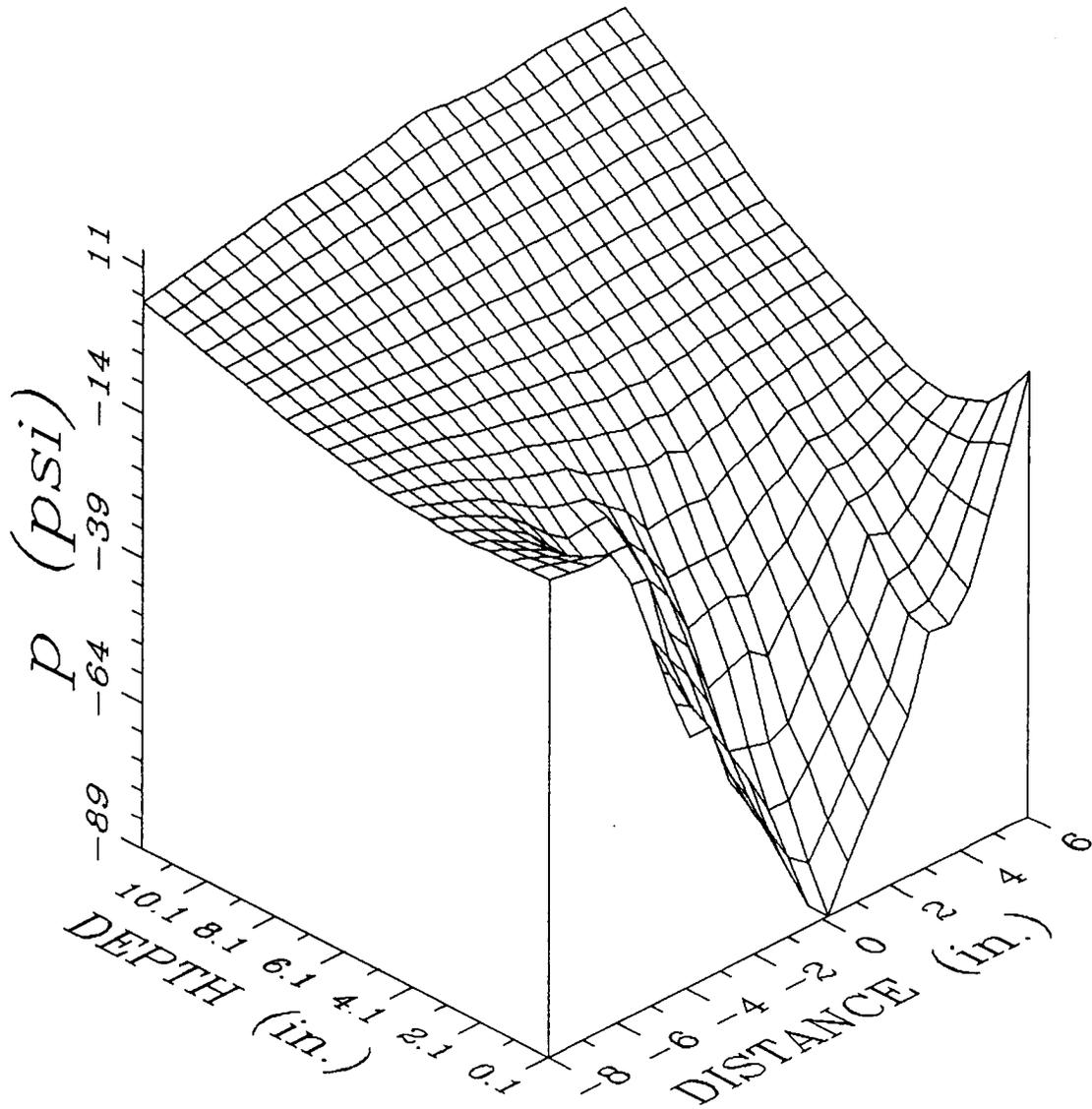
<sup>1</sup>One might ask the question why  $e$  and  $v$  have been utilized since these are elastic strains. For materials like asphalt concrete the plastic strains which develop are approximately proportional to the elastic strains (e.g., McLean and Monismith, 1974). Thus the variation in  $e$  and  $v$  also provide an indication of the variation of the corresponding plastic strains.



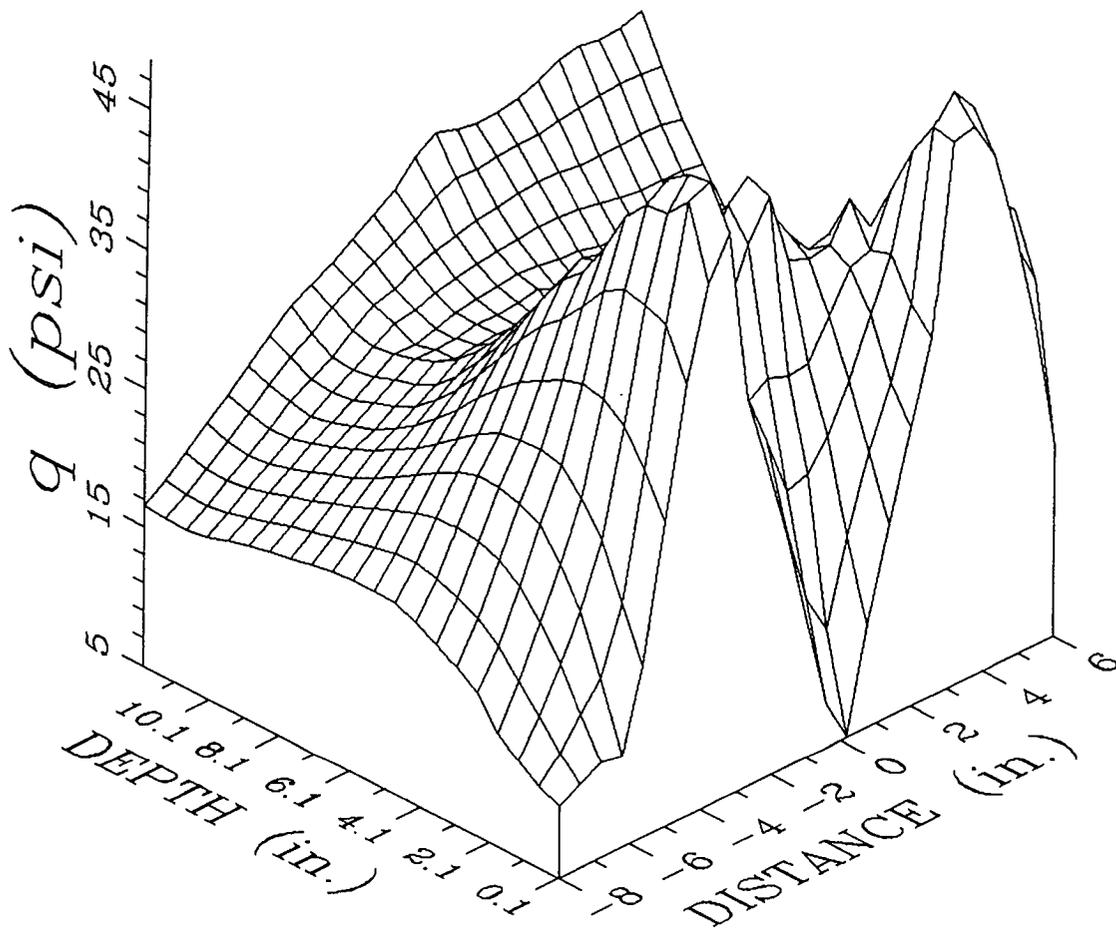
**Figure 3.2 Diagram of pavement section used to estimate values of p, q, e, and v**

**Table 3.2. Pavement section used in the determination of p, q, e, and v values shown in Figures 3.3 - 3.6.**

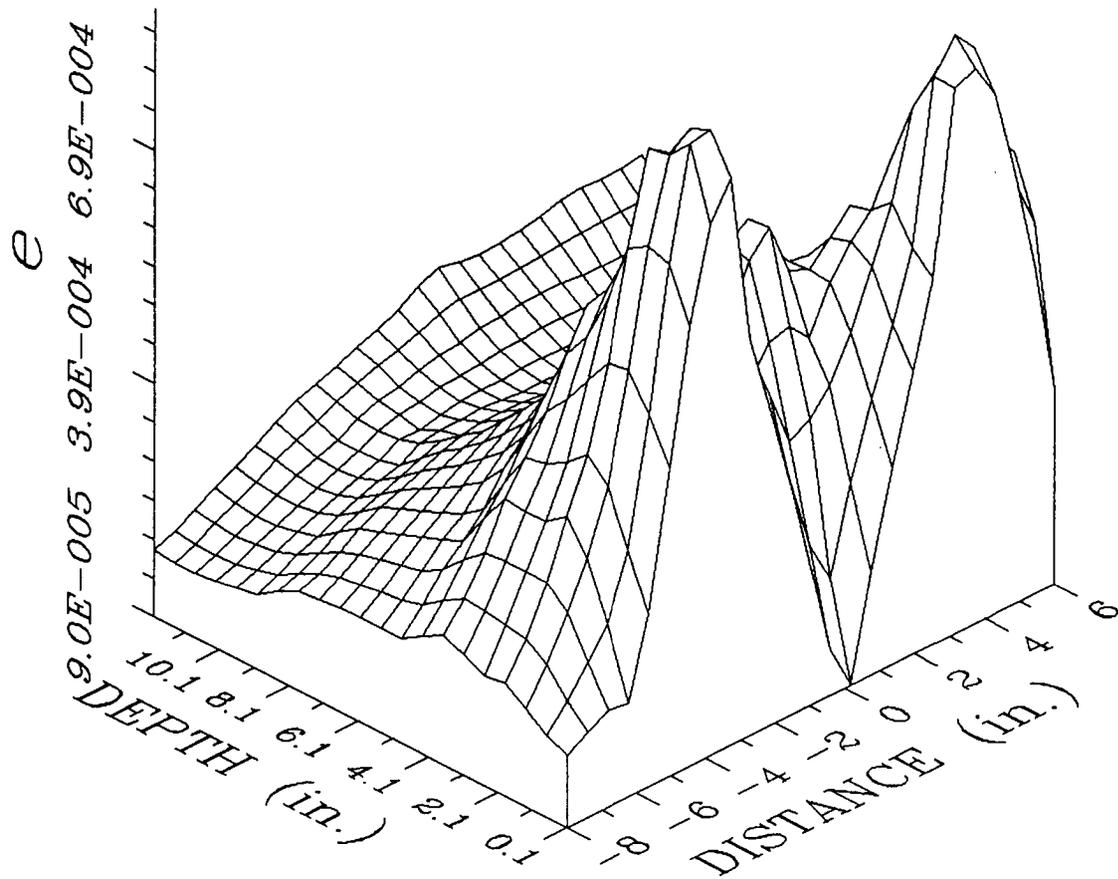
Material	Layer Number	Thickness (in.)	Poisson Ratio	Modulus (psi)
AC	1	2	.50	55,000
AC	2	2	.47	65,000
AC	3	4	.45	80,000
AC	4	4	.45	90,000
Base	5	6	.35	20,000
Subgrade	6	$\infty$	.35	10,000



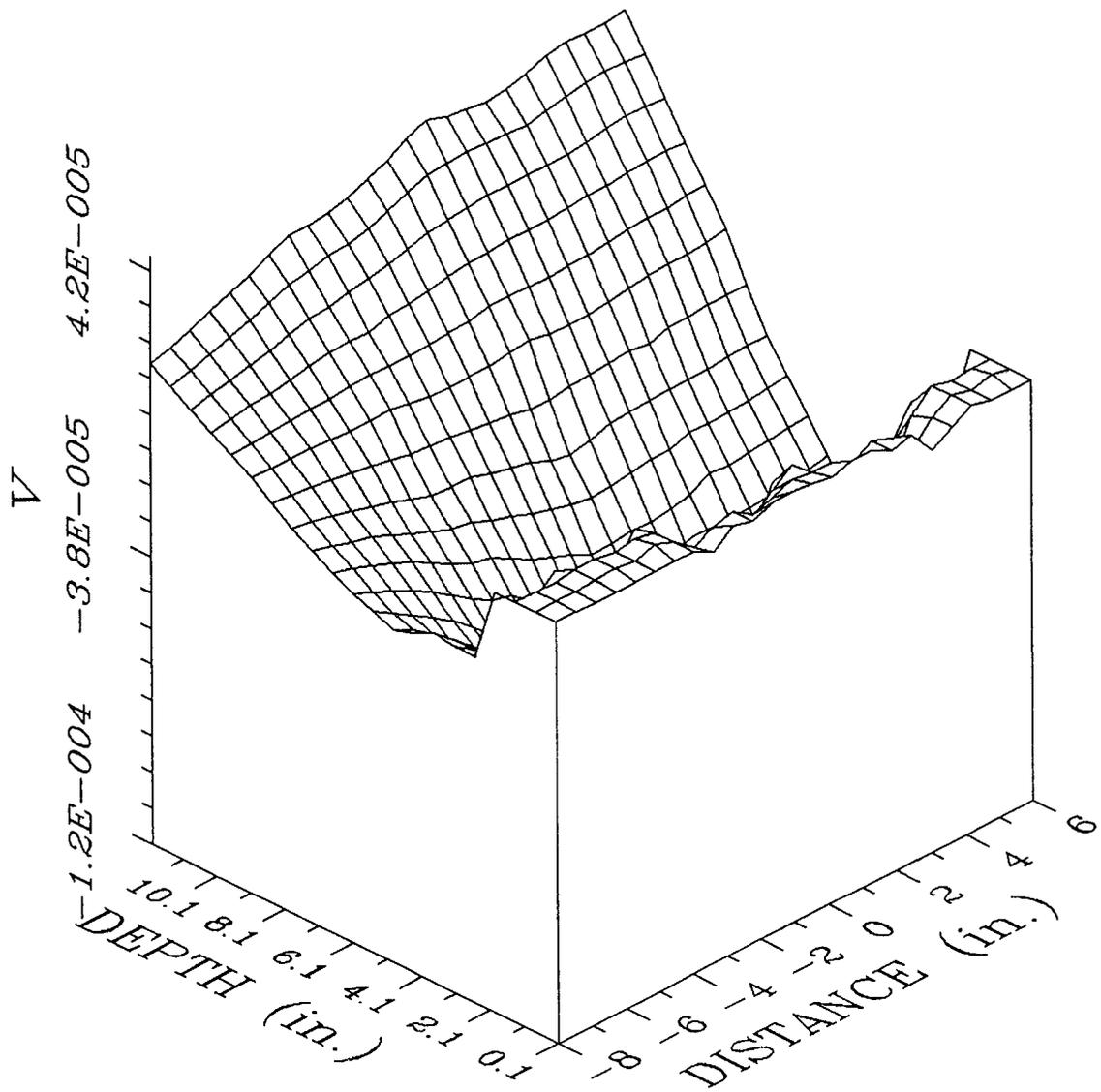
**Figure 3.3** Variation of mean normal stress,  $p$ , within the pavement section for a dual-wheel assembly



**Figure 3.4** Variation of shear stress,  $q$ , within the pavement section for a dual-wheel assembly



**Figure 3.5** Variation of shear strain,  $e$ , within the pavement section for a dual-wheel assembly



**Figure 3.6** Variation of volumetric strain,  $v$ , within the pavement section for a dual-wheel assembly

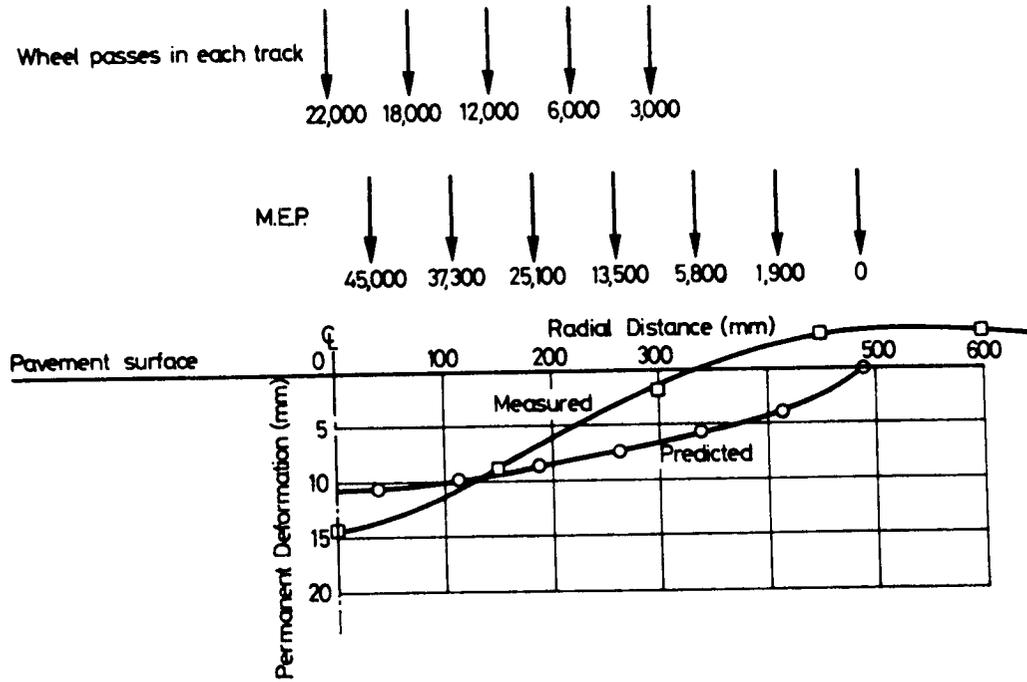
Given the complexity of the states of stress and the large number of parameters involved in the analyses, computer programs are required to define the response of multilayered systems to loads. Linear elastic, nonlinear elastic, and linear viscoelastic models have been implemented in software packages: finite element methodologies have also been used. Table 3.1 identifies computer programs used by various authors.

### **3.5 Selected Applications and Contributions**

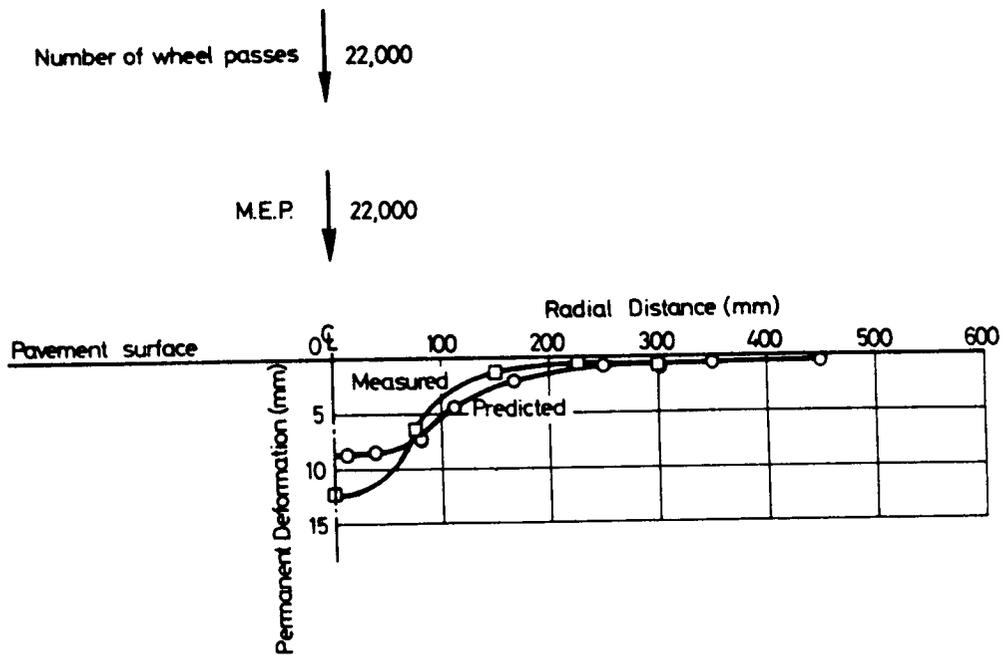
In recent years, efforts of a number of investigators have resulted in considerable improvement in the methodologies to predict rutting. These improvements have included theoretical contributions, materials characterization, consideration of the transverse distribution of traffic, distribution of temperature with depth, etc. This section summarizes some of the more important of these developments.

Brown and Bell (1977 and 1979) compared theoretically-predicted rut depths using the layer-strain approach with those measured in the Nottingham Test Track. Comparisons between predicted and measured total displacements generally indicated reasonable agreement. Figure 3.7b represents an example from their work indicating reasonable comparison. However, in the cases where upheaval occurred, the prediction was not as good (e.g., Figure 3.7a). Material properties were measured using the repeated load triaxial test. Appropriate stress states in the tensile zone of the pavement were carefully selected using the p-q approach.

Kirwan, Snaith, and Glynn (1977) presented a method for predicting permanent deformation that has been simplified for practical applications. It makes use of the layer-strain approach and a nonlinear finite element computer program. For evaluating the dynamic properties of the asphalt-concrete mixture, the authors recommended that the properties of each material be evaluated by applying the root-mean-square of the dynamic pulse loading to a creep test specimen. Thus, in the case of a sinusoidal load pattern, the equivalent static level is 61 percent of the peak to peak value. This approach offers a simple, practical method for evaluating dynamic characteristics of a material.



a. Multi-track test



b. Single track test

Figure 3.7 Measured and predicted rut profiles (After Brown and Bell, 1979)

To estimate rut development, the authors proposed a method for handling transverse distribution of loads based on the time-hardening concept described by Freeme (1973). The approach consisted of first calculating the permanent deformation as a function of number of wheel load repetitions at each specified offset distance from the centerline of the wheel track. The plastic strains were then continuously accumulated on each curve according to the past permanent deformation and number of repetitions applied corresponding to that curve. A comparison of the results of the method proposed by Kirwan, Snaith, and Glynn (1977) with measured rut depths at Nottingham for the multi-track tests, indicated that the computed rut depths were somewhat greater than the measured values, but that shapes of the measured and computed curves as a function of wheel passes were similar.

Monismith, Inkabi, Freeme, and McLean (1977) also used the layer-strain method for predicting deformation. Material properties were evaluated using the repeated load triaxial test, and pavement response was calculated using the elastic layered computer program, ELSYM. The analysis accounted for a range in axle loads and for seasonal distributions of traffic. The mean temperature of each sublayer for each month was calculated using the method proposed by Barber (1957) since temperature variation in the pavement throughout the year is an important consideration when predicting rutting in asphalt-concrete layers. The following expression was developed to relate permanent plastic strain to the stress state in the pavement:

$$\epsilon_z^p = R[\sigma_z - 0.5(\sigma_x + \sigma_y)] \quad (3.6)$$

where  $\sigma_x$  and  $\sigma_y$  are the horizontal stresses at a point in the pavement system and  $\sigma_z$  is the corresponding vertical stress;  $R$  is  $\delta(T) N^\alpha \bar{\sigma}^{\eta-1} t$  for asphalt concrete;  $\epsilon_z^p$  is the vertical permanent deformation;  $\delta(T)$  is a function of temperature;  $N$  is the number of stress applications;  $\bar{\sigma}$  is the equivalent stress ( $q$  of Equation 3.3) defined as function of the principal stresses<sup>2</sup>;  $t$  is the time of loading; and  $\alpha$  and  $\eta$  are experimentally determined coefficients. Equation 3.6 is modified from the theory of linear elasticity for use in predicting permanent deformations for any stress state. One comparison was presented

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<sup>2</sup> $\bar{\sigma} = 1/\sqrt{2} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2}$ ; for the triaxial compression test,  $\sigma_2 = \sigma_3$  and  $\bar{\sigma} = (\sigma_1 - \sigma_3)$ .

for an in-service pavement, and reasonable agreement was reported between computed and observed values of rut depth.

Meyer and Haas (1977) have developed a methodology to estimate rutting in asphalt-concrete pavements using the layer-strain approach together with mathematical models based on material properties measured in repeated load triaxial tests. In the repeated load test, both vertical and horizontal stresses were independently controlled. Statistically significant variables influencing permanent strains in the triaxial specimens were stress state, temperature, density, air voids, and number of repetitions. Rut depths were calculated in a number of typical pavements based on stresses determined by finite element techniques and verified using measured rut depths from test roads, particularly the Brampton experiment. From these results, the primary factors affecting rutting were found to be the asphalt-concrete thickness, moduli of the asphalt-concrete layer and the subgrade, and number of wheel load repetitions. To handle rutting in granular bases, the granular base was converted to an "equivalent" thickness of asphalt concrete using a substitution ratio of 1 inch of asphalt concrete for 2 inches of stone<sup>3</sup>. Meyer and Haas also found there was an optimum bituminous layer thickness to produce a minimum depth of rutting. In an actual pavement, however, it is more likely that a limiting depth of bituminous surfacing exists beyond which additional rutting is insignificant, as was found at the AASHO Road Test.

Shell researchers (van de Loo, 1978; de Hilster and van de Loo, 1977) have developed a practical procedure, utilizing layer-strain concepts, to predict rutting in the asphalt-bound layer which has been included as part of Shell's pavement design procedure.

Determining the plastic strain to use as input to Equation 3.1 requires 1) detailed information on mixture components, traffic, and the pavement environment; 2) a layered elastic analysis; and 3) data from a simple uniaxial creep test on a specimen representative of the mixture. The basic premise is that the development of deformation in an asphalt pavement is related to that occurring in a laboratory creep test (Hills, 1973). The Shell researchers have presented the results of creep tests in the form shown in Figure 3.8 and have demonstrated that mixture stiffness ( $S_{mix}$ ) and

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<sup>3</sup>Extreme caution should be exercised in converting from one type of section to another since the equivalency ratios can vary greatly with density and level of stabilization.

bitumen stiffness ( $S_{bit}$ ) are uniquely related, regardless of the temperature, since  $S_{bit}$  is a function of both time of loading and temperature. If a value of  $S_{bit}$  can be found corresponding to the field conditions leading to deformation, the resulting  $S_{mix}$  can be found from the test curve (like those shown in Figure 3.8).

In this methodology it is argued (van de Loo, 1978) that the required bitumen stiffness is the viscous component of the total stiffness,  $(S_{bit})_{visc}$ , which can be determined from:

$$(S_{bit})_{visc} = \frac{3\eta}{Nt_w} \quad (3.7)$$

where  $N$  is the total number of load applications,  $t_w$  is the loading time for one load application (seconds), and  $\eta$  is the viscosity of bitumen ( $Ns/m^2$ ).

A nomograph, provided by Shell for estimating the viscosity, requires as input the pavement temperature together with the penetration index and the temperature corresponding to 800 pen. for the asphalt. To estimate permanent deformation in the asphalt-bound layer, the following variation on Equation 3.1 is used:

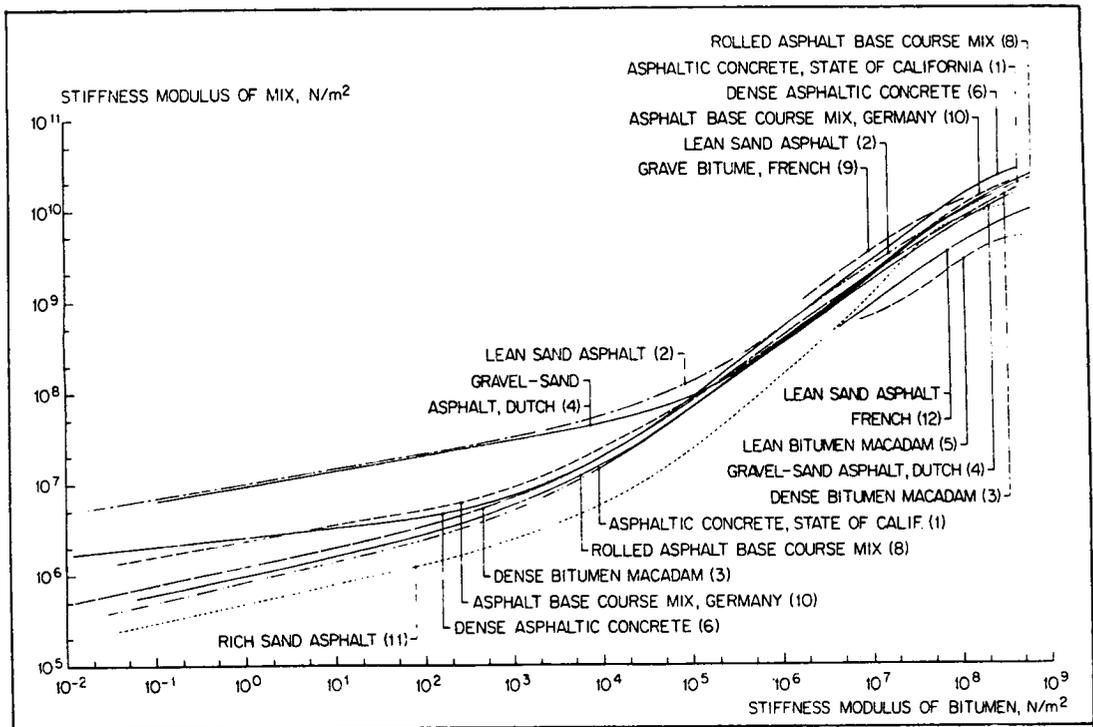
$$\Delta h = C_M * h * \left( \frac{\sigma_{av}}{S_{mix}} \right) \quad (3.8)$$

where  $\sigma_{av}$  is the average stress in the bituminous layer and determined from  $\sigma_{av} = Z\sigma_o$ ;  $h$  is the thickness of the bituminous layer;  $\Delta h$  is the change in thickness of the bituminous layer;  $S_{mix}$  is the value of the mixture stiffness for  $S_{bit}$  determined from Equation 3.7 and obtained from a curve like that shown in Figure 3.8;  $C_M$  is a correction factor;  $Z$  is a coefficient determined using layered elastic analysis; and  $\sigma_o$  is the contact stress between tire and pavement.

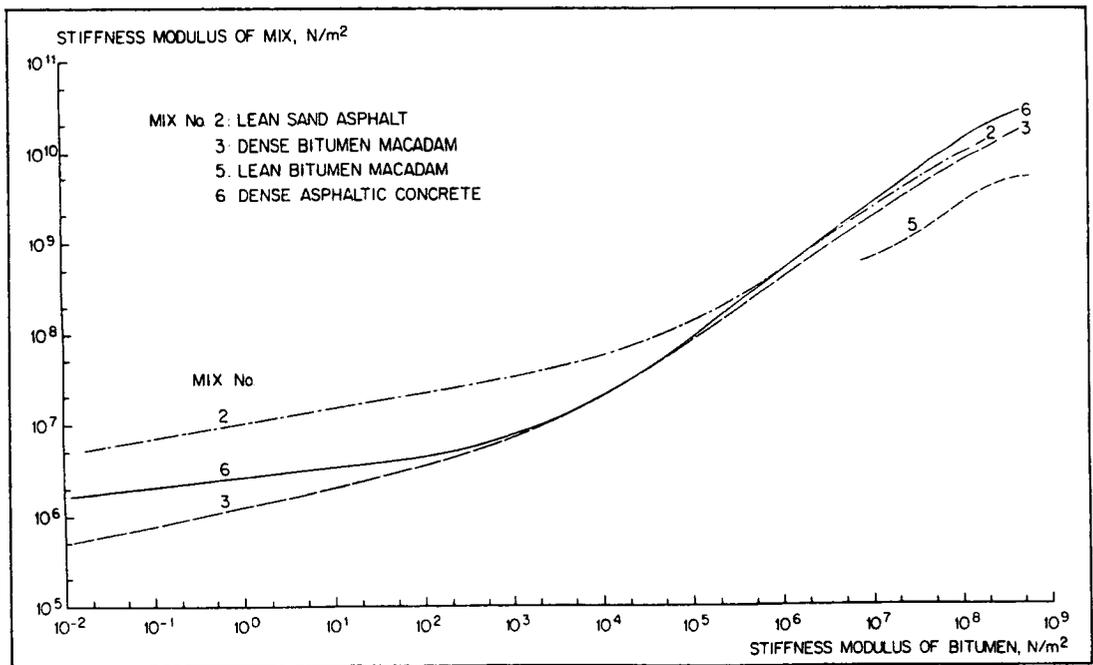
The procedure for using Equation 3.8 (Claessen et al., 1977) requires that  $\sigma_{av}$  be determined by applying a "Z" factor (determined using the BISAR program for computation of stresses and strains in a multilayer elastic system) to  $\sigma_o$ , the contact stress between tire and pavement<sup>4</sup>. Also, the method divides the bituminous layer(s) into three sublayers and calculates the deformation in each, requiring appropriate  $S_{mix}$

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<sup>4</sup>The average stress,  $\sigma_{av}$ , in a particular layer for use in Equation 3.8 can be determined by other programs for multilayer elastic systems, e.g., ELSYM (Finn et al., 1983).



**a. Range of mixtures**



**b. Representative mixtures**

**Figure 3.8 Stiffness characteristics of mixtures (After Claessen et al., 1977)**

and  $\sigma_{av}$  values for each of the sublayers.

As emphasized by Claessen et al. (1977), considerable error could result if an inappropriate  $S_{mix}$  versus  $S_{bit}$  curve is used. The  $S_{mix}$  versus  $S_{bit}$  should be determined from a creep test on a core of material, or by correlations between laboratory specimens and field cores. In the event that such data are not available, the Shell researchers have presented three creep curves representative of different mixture types: selection of one of these curves for the analysis depends on the judgment of the designer.

If the type of asphalt data required by Shell are not readily available (e.g., temperature at 800 pen.), the Bitumen Test Data Chart, BTDC, developed by Heukelom (1973) can be used to plot measured viscosities (and possibly the penetration), common measurements in the United States, and to estimate the required inputs for the stiffness nomograph (Figure 3.9).

Mahboub and Little (1988) produced a modified version of Equation 3.8 which:

- 1) Utilizes a simple power law constitutive relationship for the permanent-deformation characterization,

$$\epsilon_{vp} = a t^b \quad (3.9)$$

where  $\epsilon_{vp}$  is the viscoplastic strain,  $t$  is the time, and  $a$  and  $b$  are regression constants; and

- 2) Accounts for the nonlinearity and stress dependency within the plasticity laws as follows:

$$\Delta h = H \cdot \left( \frac{Z S_{tire}}{S_{lab}} \right)^{1.61} \cdot \epsilon_{vp}(t) \quad (3.10)$$

where  $\Delta h$  is the calculated rut depth (inches);  $H$  is the asphalt layer thickness (inches);  $Z$  is the vertical stress distribution factor (derived from layered elastic solutions);  $S_{tire}$  is the average contact pressure;  $S_{lab}$  is the stress level at which the creep test is conducted (psi); and  $\epsilon_{vp}(t)$  is the viscoplastic strain trend of the mixture measured by the creep test (in/in).

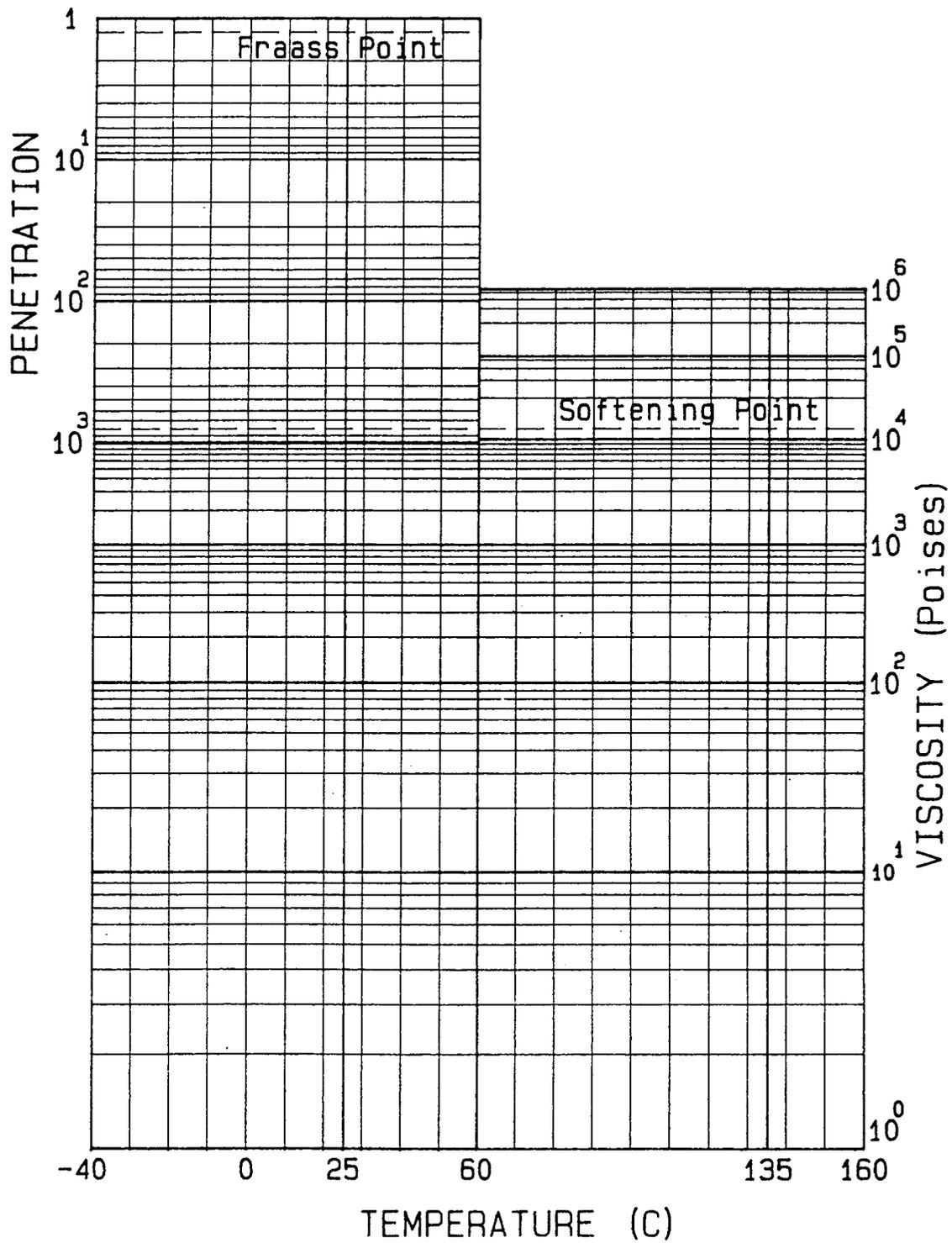


Figure 3.9 Bitumen test data chart (After Heukelom, 1973)

Barksdale and Leonards (1967) utilized an approach incorporating linear viscoelastic theory, repeated stationary loading at the pavement surface, and material properties obtained from the repeated load triaxial test. Comparison of the calculated rut depth with the measured value after 100,000 repetitions for a representative section from the AASHO Road Test indicated that the viscoelastic analysis resulted in a slightly smaller rut depth than actually measured (approximately a factor of 0.8). Comparisons of computed and measured recoverable deflections indicated similar variation.

Battiato, Ronca, and Verga (1977) developed a solution for a two-layer incompressible viscoelastic pavement system. The asphalt-bound layer was characterized by means of a creep compliance with a viscous element in series, and the subgrade was assumed to be elastic. The creep compliance was evaluated from unconfined creep tests. The researchers noted that the predicted spatial distribution of permanent deformations arising from the viscous element exhibited features which might prove useful in comparing computed values with actual field experiments.

Huschek (1977) proposed the use of linear viscoelastic theory for calculating rutting in a layered pavement system. A Maxwell model was used to represent the response of the asphalt-bound layer. In the model, the viscosity of the dashpot was assumed to increase with increasing numbers of load repetitions. Both conventional and cyclic unconfined creep tests were used to derive material characteristics. Huschek suggested that the specimens should be confined to better define the influence of the internal friction of the aggregate. The method includes provisions for the temperature distribution in the pavement, transverse distribution of traffic, and vehicle speed. Only the bituminous layers were considered to have time-dependent material properties.

Verification was accomplished by comparing measured and calculated rut depths for a test road near Zurich. Creep tests were run on cores from each of the five test sections. The comparison between measured and calculated rut depths was done on a relative basis: traffic density variations in relation to temperature were not well enough known to calculate exact values of rut depth. The relative comparisons found reasonably good agreement between calculations and measurements.

Kenis (1977, 1982) has presented an overall framework for a viscoelastic analysis system

(VESYS) which consists of four major interactive models, termed primary response, general response, damage, and performance models. The primary response model is a probabilistic linear viscoelastic solution for the mean and variance of the time dependent stress, strain, and deflection at prescribed positions in a layered viscoelastic system.

The rut depth predictive submodel (part of the damage model) is based upon the fundamental assumption that rut depth is proportional to the log of load repetitions. The permanent strain behavior of all materials in the pavement system is modeled by an equation of the form:

$$\epsilon_p(N) = \epsilon \mu N^{-\alpha} \quad (3.11)$$

where  $\alpha$  and  $\mu$  are material (regression) constants and  $\epsilon$  is the peak haversine load strain for a load pulse with 0.1 sec load duration at the 200th repetition.

In the VESYS methodology, the relationship between total, resilient, and permanent strains is:

$$\epsilon_p(N) = \epsilon - \epsilon_r(N) \quad (3.12)$$

where  $\epsilon_r$  is the resilient (recoverable) strain. Using this assumption, expressions for the resilient compliance,  $D_r$ , and resilient modulus,  $E_r$ , are as follows:

$$D_r(N) = D(1 - \mu N^{-\alpha}) \quad (3.13)$$

In this formulation, the two main material characterization parameters are the  $\alpha$  and  $\mu$  variables. In this case:

$$E_r(N) = \frac{E N^\alpha}{N^\alpha - \mu} \quad (3.14)$$

- 1)  $\alpha > 0$  signifies a stress hardening material;
- 2)  $\alpha = 0$  signifies a constant (elastic) material; and

- 3)  $\alpha < 0$  signifies a stress softening material.

For bituminous materials, the  $\alpha$  and  $\mu$  parameters are functions of temperature.

Knowledge of the total material response and the  $\alpha$  and  $\mu$  parameters allows for the separation of response values into their resilient or permanent (plastic) components. For rut-depth measurements, permanent strains are used in the analysis. To predict total pavement rutting, use is made of a "system" model, determined from evaluating the combined layer responses. It is similar to the individual layer models as shown below:

$$Y_p(N) = Y\mu_{sys}N^{-\alpha_{sys}} \quad (3.15)$$

where  $Y_p(N)$  is the systems incremental permanent deformation per pulse;  $Y$  is the systems general haversine deflection at peak loading; and  $\mu_{sys}$  and  $\alpha_{sys}$  are system permanent-deformation properties which are determined internally to the computer program. Integrating this equation with respect to  $N$  yields the accumulated rut depth for a given season.

Thrower (1977) and Thrower et al. (1986) have examined a number of different approaches for the determination of permanent deformation including viscoelastic methodologies. They have suggested that "separative" methods, based on deriving the permanent deformation from experimentally-determined material deformation properties, in association with a stress distribution determined separately and not considering the permanent deformation (e.g., Monismith et al., 1977), may lead to "serious distortions" in assessing the role of the various layers in contributing to permanent deformation.

In the 1986 report, Thrower et al. outlined a method of analysis using a linear viscoelastic model for material behavior to provide values for the long-term irrecoverable surface displacement due to a given load history. They have also extended the analysis to nonlinear materials using an incrementally linear formulation. Their solutions include provision for both circular and strip loads, the latter being considered to be more appropriate since it can provide a residual displacement field of the form expected with traffic loading. An interesting general conclusion from their analyses is that the permanent displacement depends only on viscous parameters and not at all on

elastic properties.

As a part of these developments, Thrower et al. developed a finite element program which included all the features required to obtain a direct solution for permanent displacements. Figure 3.10 illustrates a comparison of the results of their computations as well as a comparison with results developed by Bjorklund (1984).

These results suggest that viscoelastic considerations together with the use of finite element methodology may provide an improved procedure to estimate the development of permanent deformation under repeated trafficking. Interestingly, Goacolou (1987) has also presented such an approach. The input parameters for his model, incorporated in a program termed CASTOR, include: 1) master curves of the asphalt-bound materials; 2) laws of flow of the asphalt-bound materials; 3) vertical temperature distribution; 4) vehicle velocities; 5) axle loads; 6) transverse positions of axle passages; and 7) elastic moduli and Poisson's ratios of the other materials comprising the pavement structure. This methodology has been used to simulate the permanent deformations which develop in the Laboratoire Central des Ponts et Chaussées (LCPC) rutting tester (see Section 4). General results for the analysis are illustrated in Figure 3.11.

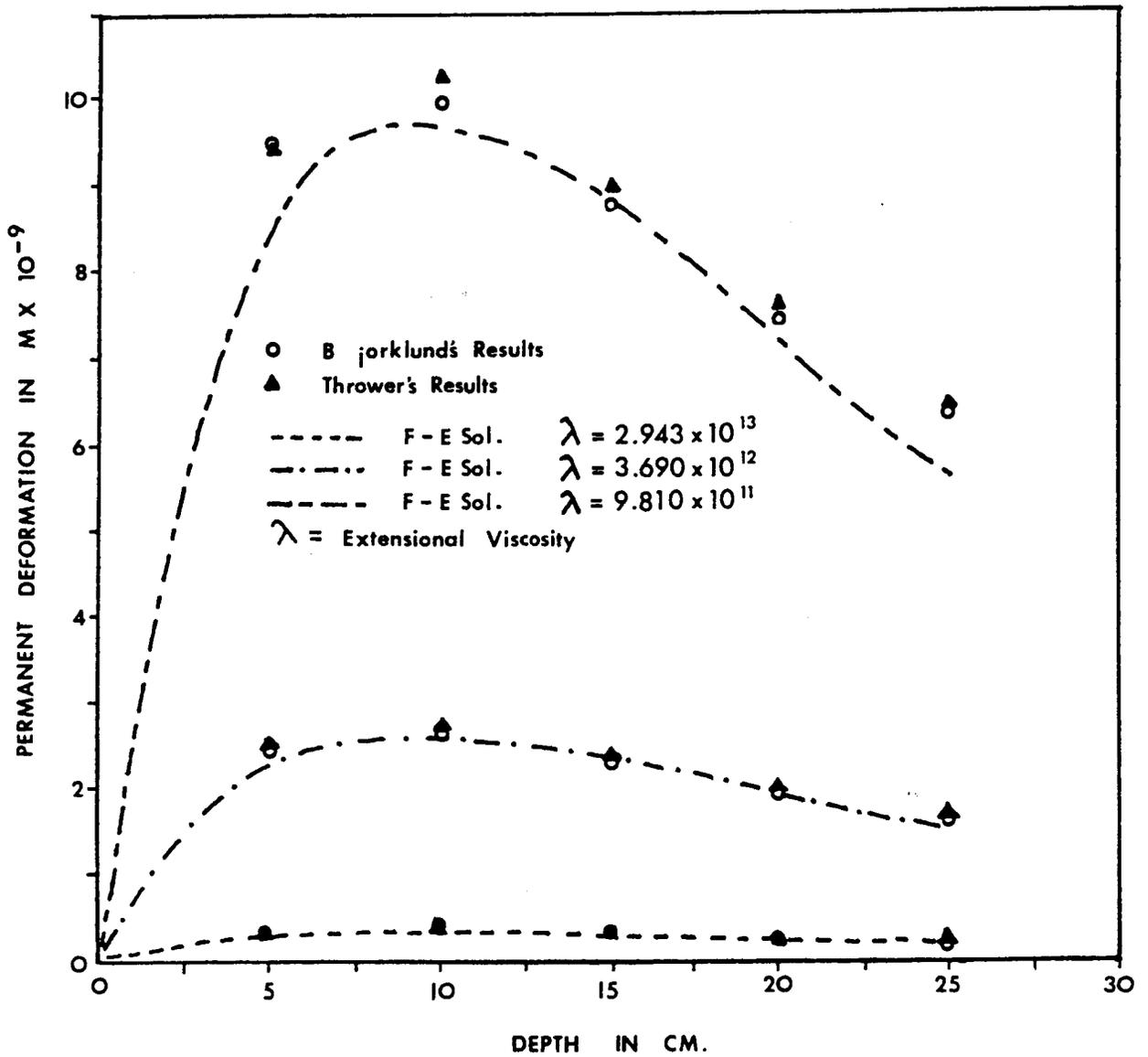
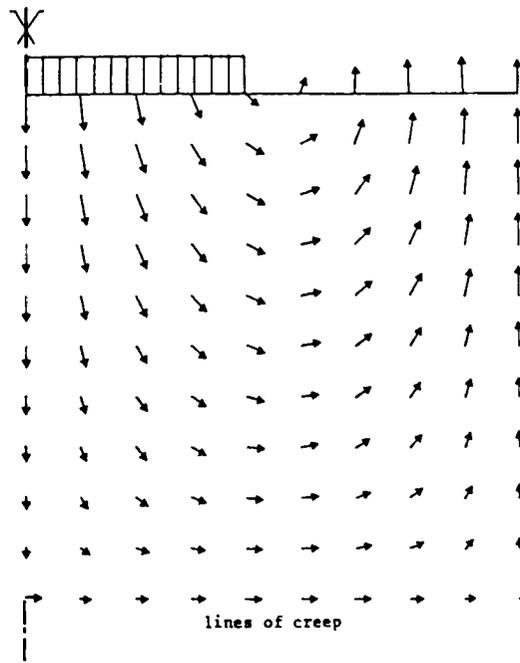
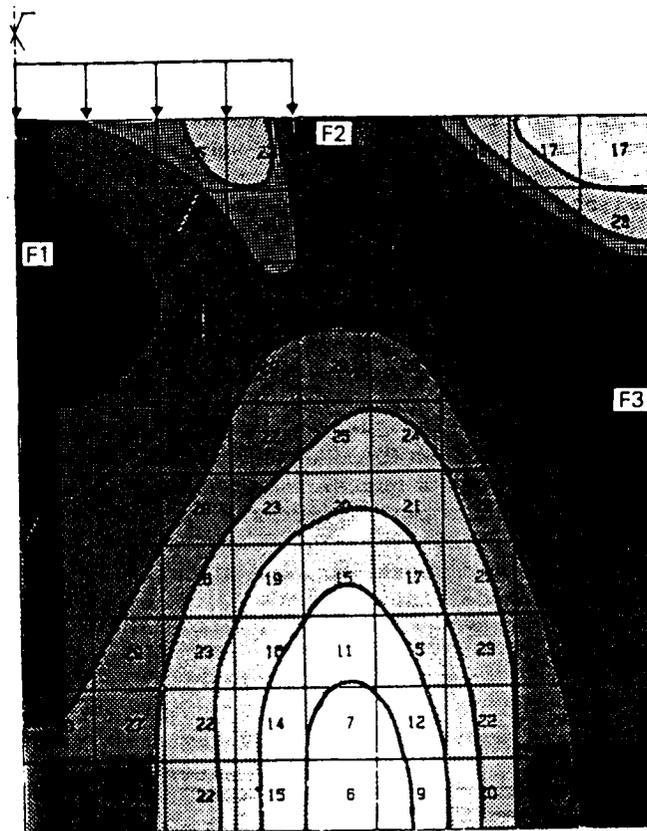


Figure 3.10 Comparison of results from finite element program with linear viscoelastic solutions (After Thrower et al., 1986)



a. Directions of permanent displacements



b. Permanent shear strain rating

Figure 3.11 Computed displacements and shear strains in the LCPC rutting test  
(After Goacolou, 1987)

# 4

## Materials Testing

The development of predictive methods or models requires suitable techniques not only for calculating the response of the pavement to load but also for realistically characterizing the materials. Table 3.1 identifies a wide range of models representative of the permanent-deformation behavior of asphalt mixtures. They fall into three general categories: 1) empirical regression equations, 2) "typical" plastic strain laws, and 3) functional equations directly based on laboratory test results. In all cases laboratory tests are performed to determine the representative parameters. From an analysis of these models of permanent deformation, the primary factors affecting rutting are found to be temperature, number of load applications or time of loading, mixture properties, and the state of stress.

In some of the models, the state of stress is identified as the axial compressive stress ( $\sigma_1$ ) and the horizontal stress ( $\sigma_2$ ). In fact, however, rutting appears to be more directly related to the shear stress, which can be obtained from  $(\sigma_1 - \sigma_2)$ . Célard (1977) emphasizes, based on results of dynamic creep tests, the important effect of shear stress on the rate of permanent deformation (Figure 4.1). For example, in Célard's tests, increasing the shear stress from 0.1 MPa to 0.25 MPa (with the normal stress at 0.1 MPa) increased the rate of permanent deformation from 0.1 to 10 (a 100-fold increase). On the other hand, varying the normal stress from 0.1 to 0.25 MPa (at constant shear stress of 0.1 MPa) did not significantly change the rate of permanent deformation. Similar conclusions can be derived from the work of Brown and Bell (1977) (Figure 4.2).

The overall objective of materials testing should be to reproduce as closely as practical in situ pavement conditions, including the general stress state, temperature, moisture, and general condition of the material. Types of tests presently used to characterize the permanent deformation response of pavement materials include the following:

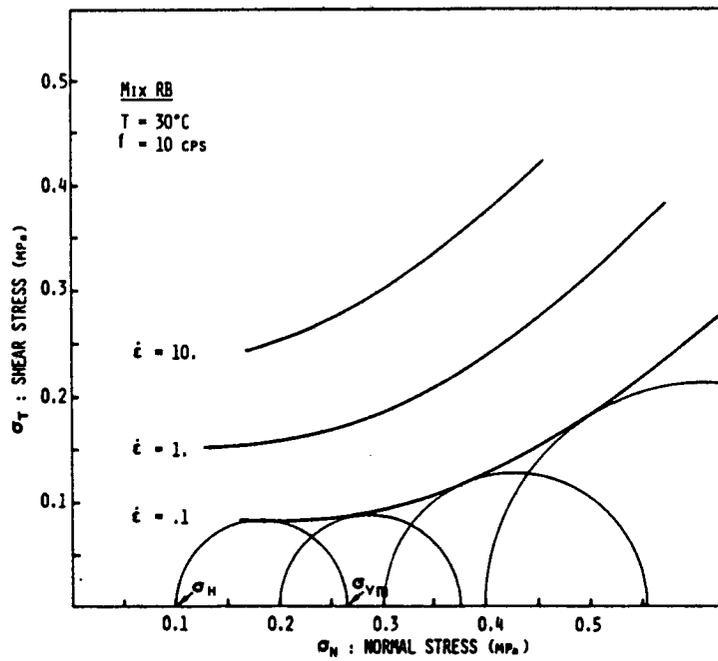


Figure 4.1 Isocreep curves (After Célard, 1977)

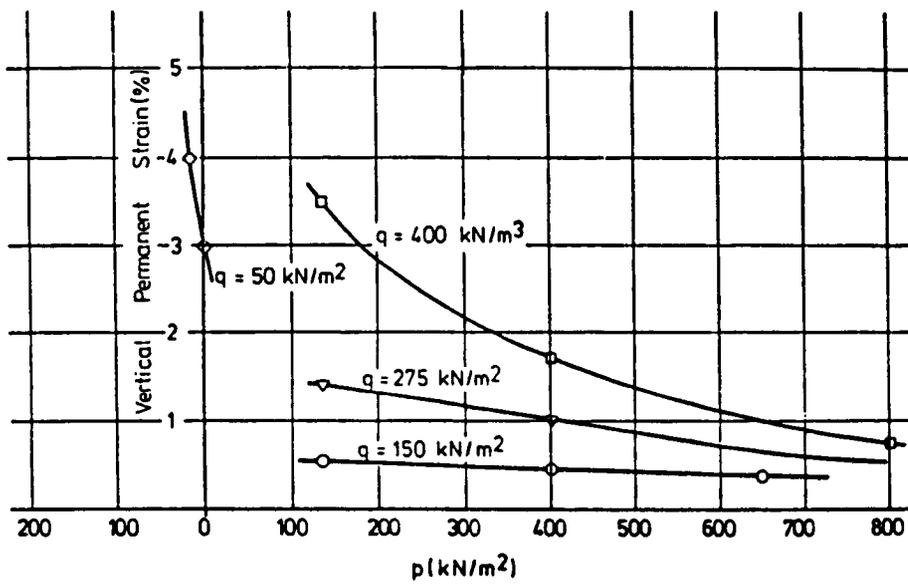


Figure 4.2 Permanent strain in dense bitumen macadam after 1000 cycles as a function of stress conditions (After Brown and Bell, 1977)

- 1) Uniaxial stress tests: unconfined cylindrical specimens in creep, repeated, or dynamic loading;
- 2) Triaxial stress tests: confined cylindrical specimens in creep, repeated, or dynamic loading;
- 3) Diametral tests: briquet specimens in creep or repeated loading; and
- 4) Wheel-track tests: slab specimens or actual pavement cross-sections.

These tests are used to evaluate the elastic, viscoelastic, plastic, and shear-strength parameters of asphalt concrete.

#### 4.1 Uniaxial and Triaxial Creep Tests

The creep test (unconfined or confined) has been used to measure mixture characteristics for a variety of predictive methods. Among its users have been researchers at the Shell Laboratory in Amsterdam. They have conducted extensive studies using the unconfined creep test as a basis for predicting rut depth in asphalt concrete (van de Loo, 1976; Uge and van de Loo, 1974; van de Loo, 1974; and Hills, 1973). To obtain good comparisons between relative rut depths observed in a test track and those calculated using creep test data, van de Loo (1974) found that the creep test must be performed at relatively low stress levels--within the linear range of the materials. He later concluded (van de Loo, 1976) that tests performed at 15 psi gave acceptable results. The need to use a stress level within the linear range has been attributed to the fact that the loading time in situ is small compared to the loading time in the creep test.

A detailed description of the compressive creep test for asphalt mixtures has been given by van de Loo (1978). An asphalt concrete specimen with flat and parallel ends is placed between two hardened steel platens, one of which is fixed and the other movable. A constant load is placed on the movable platen, and the deformation of the specimen is measured with LVDTs as a function of time. The test temperature is kept constant. A system that can be used for creep testing is shown in Figure 4.3.

Results of the creep test, when expressed as relative deformation (measured change in height divided by original height), are independent of the shape of the specimen and of the ratio of height to diameter, provided the specimen's ends are parallel, flat, and well lubricated. Lubrication is necessary to prevent the platens from laterally constraining the specimen, to provide a uniform stress distribution, and to prevent barrelling (van de Loo, 1978). Strain, measured as a function of the loading time ( $t$ ) at a fixed test temperature ( $T$ ), is the usual test output.

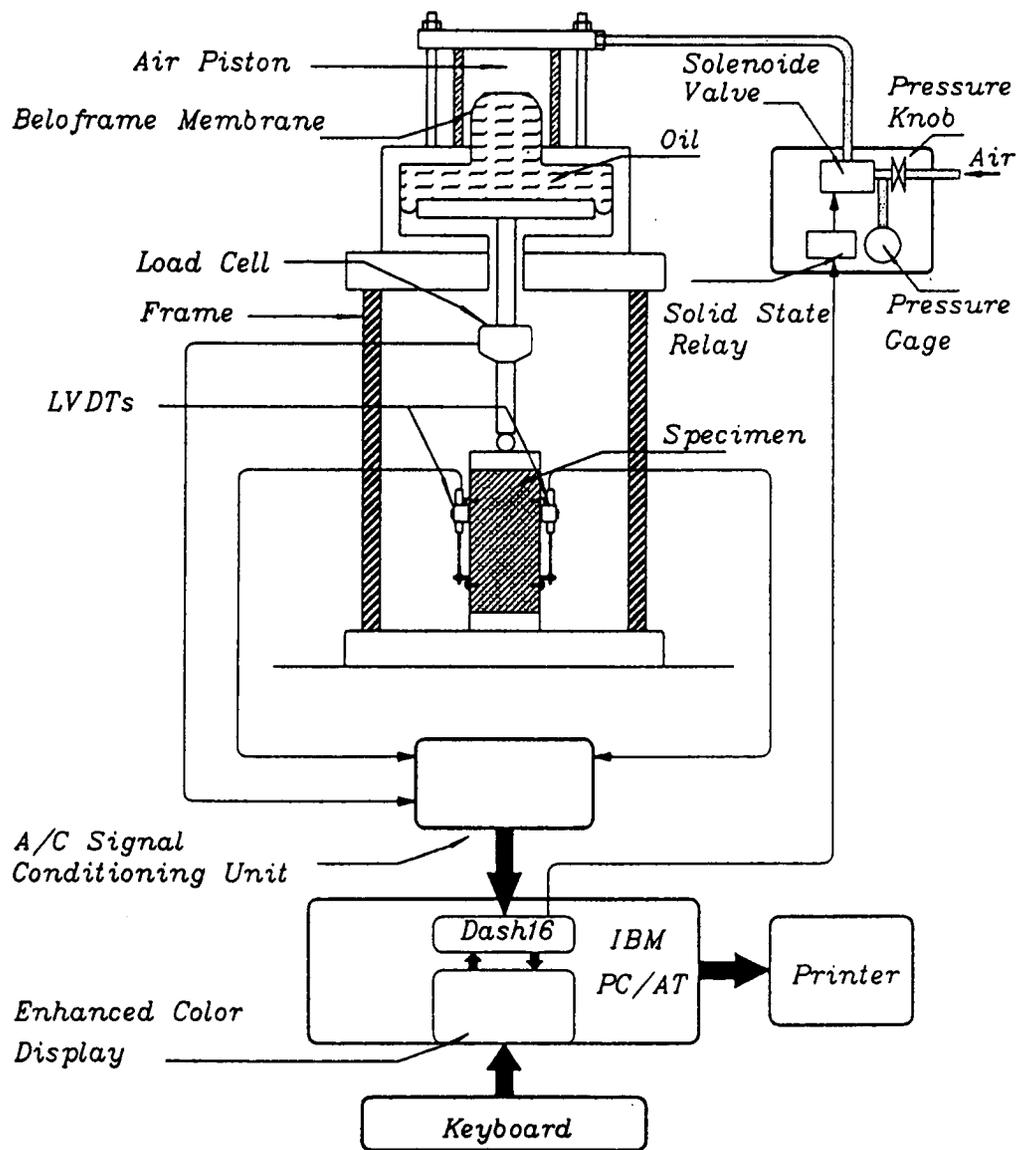


Figure 4.3 Test system for creep and repeated load testing

This test is the most widespread one for determining material properties for predictive analyses because of its simplicity and the fact that many laboratories have the necessary equipment and expertise.

## 4.2 Uniaxial and Triaxial Repeated Load Tests

A variety of loading systems has been used to measure mixture response in repeated loading, ranging from relatively simple mechanical or pneumatic systems to more complex electro-hydraulic systems. Pneumatic systems, such as shown in Figure 4.3, have probably been the most suitable for routine testing. More sophisticated systems generally include a testing chamber that permits careful control of temperature as well as the application of repeated load confining pressure coordinated with the vertical repeated load. Such equipment may have the following capabilities:

- 1) Applying repeated axial and lateral stress pulses of any desired shape, in phase with one another, with pulses ranging from 0.01 to 1.0 second;
- 2) Applying the axial stress in either tension or compression;
- 3) Incorporating rest periods between stress pulses, ranging from near zero to several seconds; and
- 4) Controlling temperature within a tolerance of 0.5°F.

The permanent vertical and horizontal strains or deformations are easily measured by LVDTs on the axial ram and lateral deformation gauges on the specimen. Resilient modulus, plastic (permanent) strain, and Poisson's ratio, as functions of the numbers of load repetitions, can be ascertained from these measurements. The important point to remember is that pulse shape and duration can greatly influence the measurements: they must duplicate as closely as possible conditions existing in the actual pavement. Estimates of in situ pulse duration and shape, as functions of vehicle speed and depth within the pavement, are available from published charts (Barksdale, 1971).

Repeated load tests appear to be more sensitive to mixture variables than creep tests. For example, Barksdale and Miller (1977) reported that, on the basis of Shell creep tests, an increase in the asphalt content of a particular mixture from 4.5 to 5.5 percent should not have a significant effect on the rut depth. Results of repeated load triaxial tests on the same mixture, however, indicated that such an increase in asphalt content could increase the rut depth by 16 percent (Figure 4.4). On the basis of extensive testing, Barksdale (1972) concluded that repeated load triaxial tests appear to better

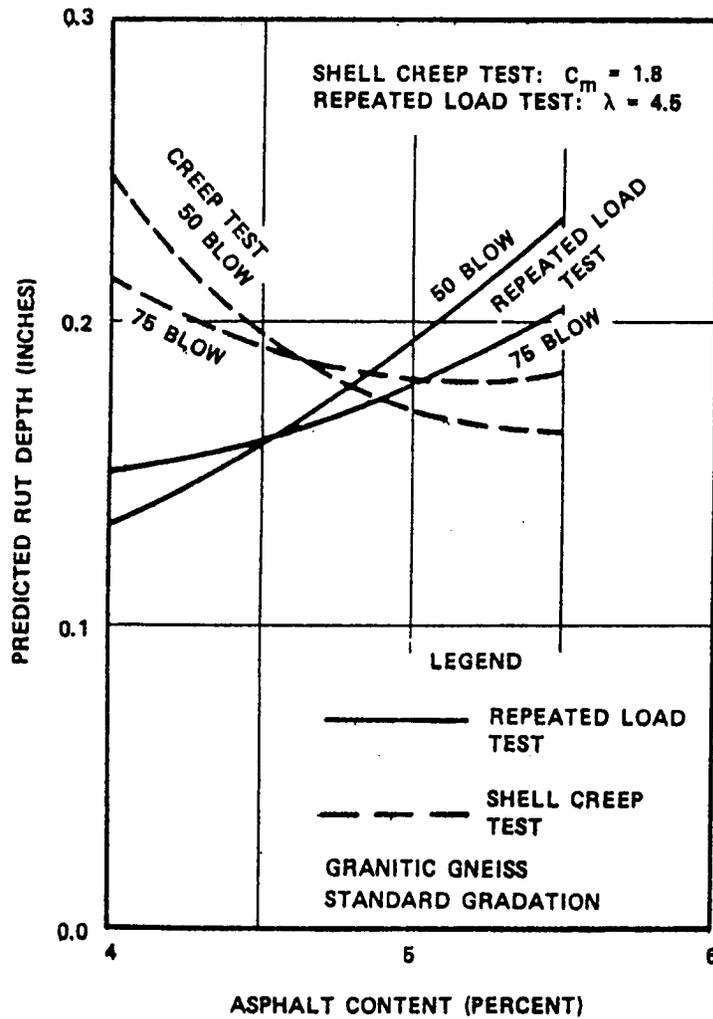


Figure 4.4 Comparison of rut depth predictions from repeated load and creep tests - black base mixture (After Barksdale and Miller, 1977)

measure rutting characteristics than the creep test, interpreted using Shell procedures.

Similar conclusions were drawn by Monismith and Tayebali (1988). They compared the response of three mixtures containing conventional and modified binders (AR2000, AR8000, and AR2000 + 20 percent carbon black microfiller) under both creep and repeated loading (Figure 4.5). For creep loading at 100°F (37.8°C) and a confining pressure of 30 psi, differences among the mixtures were not discernable. There were, however, differences in the repeated load data. This study suggests that the repeated loading test may be a more appropriate tool with which to investigate the permanent-deformation characteristics of asphalt mixtures than the creep test. Further studies should be conducted to substantiate this finding.

### **4.3 Triaxial Dynamic Tests**

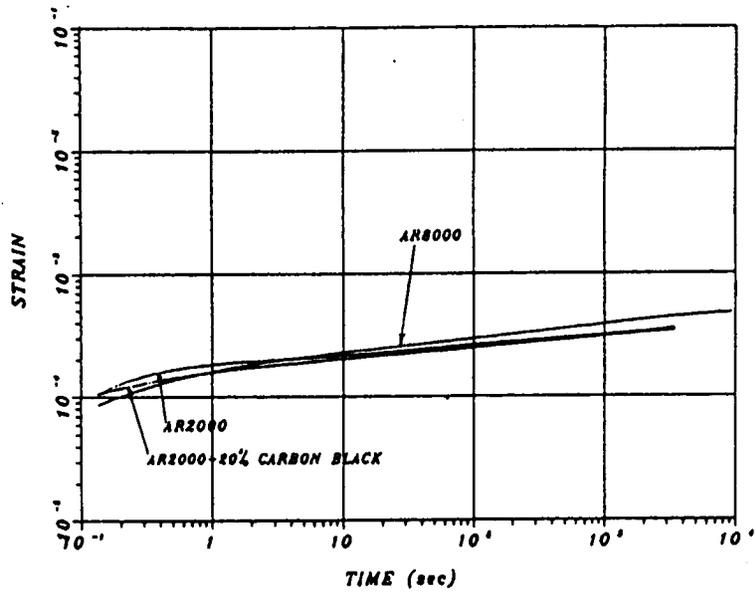
This test was used by Francken (1977) in the determination of dynamic and creep properties of cylindrical asphalt-concrete specimens. A constant lateral pressure was used, and the vertical pressure was varied sinusoidally over a range of frequencies. Equipment for executing this test has typically been operated under feedback closed loop control. Although costs for sophisticated control have been relatively high in the past, recent developments in microcomputers and data acquisition analogue/digital boards have made this equipment relatively inexpensive and accessible (Sousa and Chan, 1991; Chan and Sousa, 1991).

Triaxial dynamic tests permit the determination of additional fundamental properties such as the dynamic modulus and the phase angle as functions of the frequency of loading, the number of load cycles, and temperature. Used to characterize the load response of linear viscoelastic materials, the dynamic modulus is simply the ratio of peak stress to peak strain while the phase angle, a measure of damping, represents the amount by which strain response lags the applied stress.

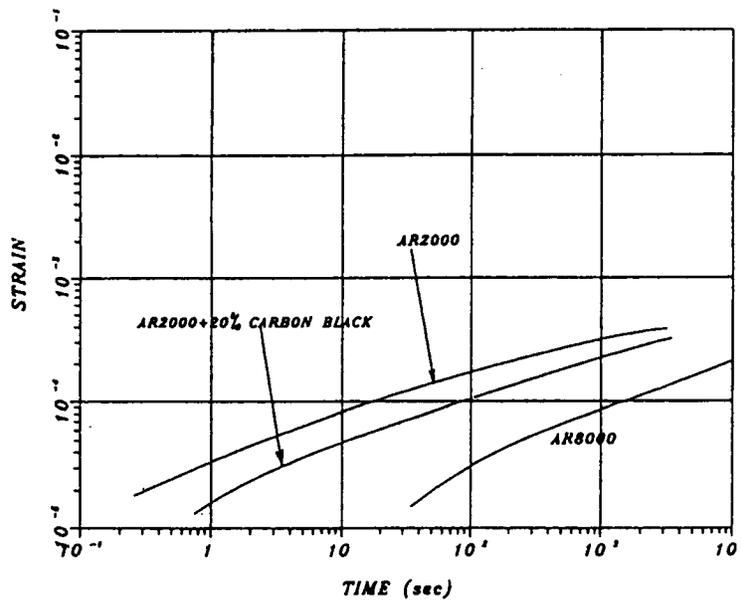
### **4.4 Evaluation of Uniaxial and Triaxial Tests**

Uniaxial and triaxial tests (creep, repeated, and dynamic) have been extensively used for the following reasons:

- 1) Relatively uniform states of stress are applied if proper care is taken in lubricating the ends of the specimens;
- 2) A wide range of stress states can be created by varying axial and confining



a. Creep



b. Repeated loading

Figure 4.5 Comparison of three mixtures in triaxial compression tests at 100°F, 30 psi confining pressure

pressures. Some of these states of stress include shear components, and most of the states of stress that are encountered within pavements can be duplicated; and

- 3) The tests are relatively easy to implement.

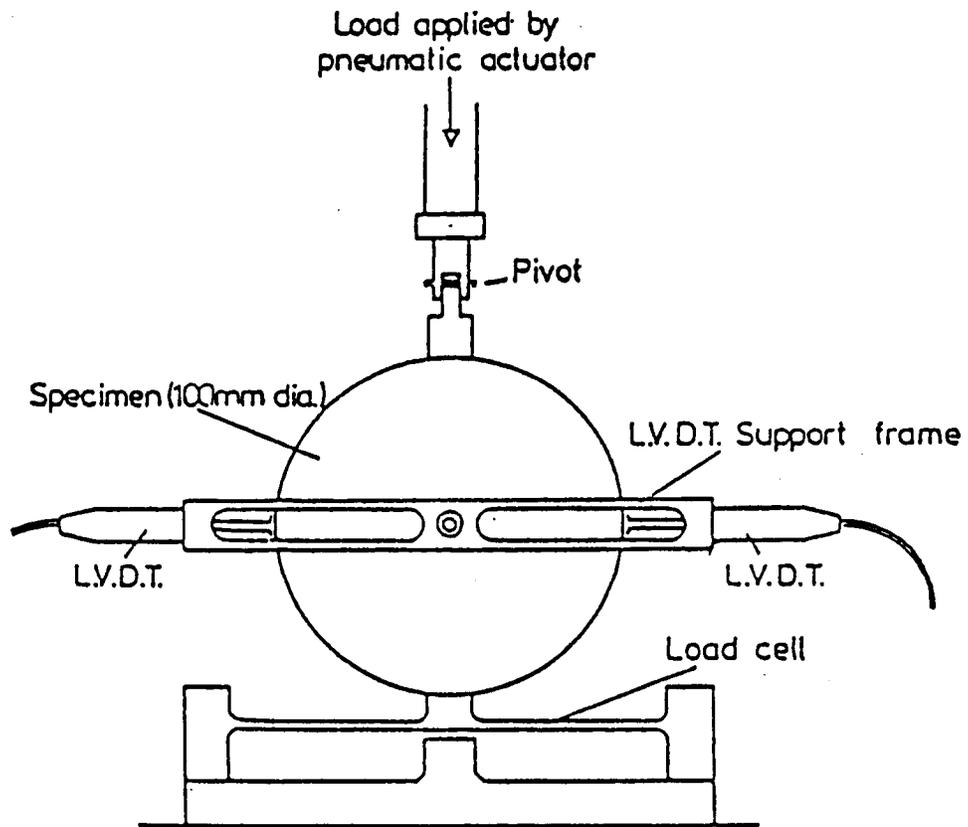
On the basis of a study of computed stresses in idealized pavements, Thrower (1978), however, suggested that limitations inherent in the triaxial test--specifically the relative values of the stress invariants that can be achieved--make it undesirable to rely solely on triaxial testing to define material behavior for predicting pavement performance. He further suggested that another test should be used to investigate properties of materials composing the upper bound layers because the states of stress encountered in upper layers cannot be duplicated in triaxial testing. This test should be able to develop states of stress with relatively higher shear components than the triaxial test.

#### **4.5 Diametral Tests**

An alternative test for measuring the stiffness of asphalt mixtures is the diametral (indirect-tension) device originally described by Schmidt (1972). Several versions of this device have recently been used: all operate in a manner similar to that shown in Figure 4.6. With this loading, most of the specimen is in tension along the vertical diameter in line with the load. Thus, the stiffness or resistance to load is largely a function of the asphalt binder, and the aggregate has less influence than in triaxial testing. Accordingly, the diametral test may be better suited to the repeated load testing associated with modulus measurements than to the longer time periods associated with creep measurements. Without confinement, large loads tend to unrealistically deform the specimen and, if the stress or temperature is too high, creep deformations will accelerate with time.

Diametral testing does not seem promising for permanent-deformation characterization for two major reasons:

- 1) The state of stress is nonuniform and strongly dependent on the shape of the specimen (Sousa, 1990). At high temperatures or large loads, permanent deformation produces changes in the specimen shape that significantly affect both the state of stress and test measurements.
- 2) During the test, the only relatively uniform state of stress is tension along the vertical diameter of the specimen. All other states of stress are distinctly nonuniform. It has been recognized that shear stresses contribute significantly to rutting and that laboratory tests must duplicate



**Figure 4.6 Schematic of repeated load diametral test  
(After Brown and Cooper, 1984)**

in situ conditions. Further, shear stresses cause nonlinear behavior in the permanent-deformation response of asphalt concrete (Célar, 1977).

Because a nonuniform field of shear stresses is present in diametral specimens, deformation measurements cannot be related to a specific stress level.

Khosla and Omer (1985) compared rutting predictions obtained from uniaxial creep tests and from diametral creep tests with values measured from an in-service pavement. Permanent-deformation parameters ( $\mu$  and  $\alpha$ ), determined from incremental creep tests using both test methods, were used as input to the VESYS computer program. Use of mechanical properties determined by diametral testing almost always resulted in overestimates of pavement rutting.

#### **4.6 Torsion Shear Tests on Hollow Cylindrical Specimens**

Controlled changes in the magnitude and direction of principal stresses on a material element are extremely difficult to reproduce in the laboratory. In most equipment, e.g., that used for triaxial or plane strain loading, principal stresses are fixed in one direction, and only an interchange of principal stress directions can take place. Rotation of the principal stress axes can only be accomplished in equipment in which shear stresses can be applied to the specimen surfaces. A laboratory simulation of principal stress rotation involves subjecting hollow cylindrical specimens to axial load ( $W$ ) and torque ( $M_T$ ), about a central axis, and to internal and external radial pressures,  $p_i$  and  $p_o$ , respectively (Figure 4.7). Due to the symmetry of the hollow cylindrical specimen, the normal and shear stresses are uniformly applied.

This type of apparatus (Figure 4.8) was used by Sousa (1986) to determine dynamic properties of asphalt concrete under axial and torsional loads. However, the apparatus also offers valuable capabilities in determining permanent-deformation characteristics of asphalt concrete under three-dimensional states of stress with reversal of shear stresses. Although the equipment is quite sophisticated and much too complex to be standardized for routine applications, it is very useful as a research tool.

#### **4.7 Simple Shear Tests**

Simple shear tests have been widely used in the measurement of soil properties. Their increasing use stems from both 1) a greater awareness of the importance of stress-strain anisotropy in geotechnical problems and 2) the simplicity of simple shear testing relative

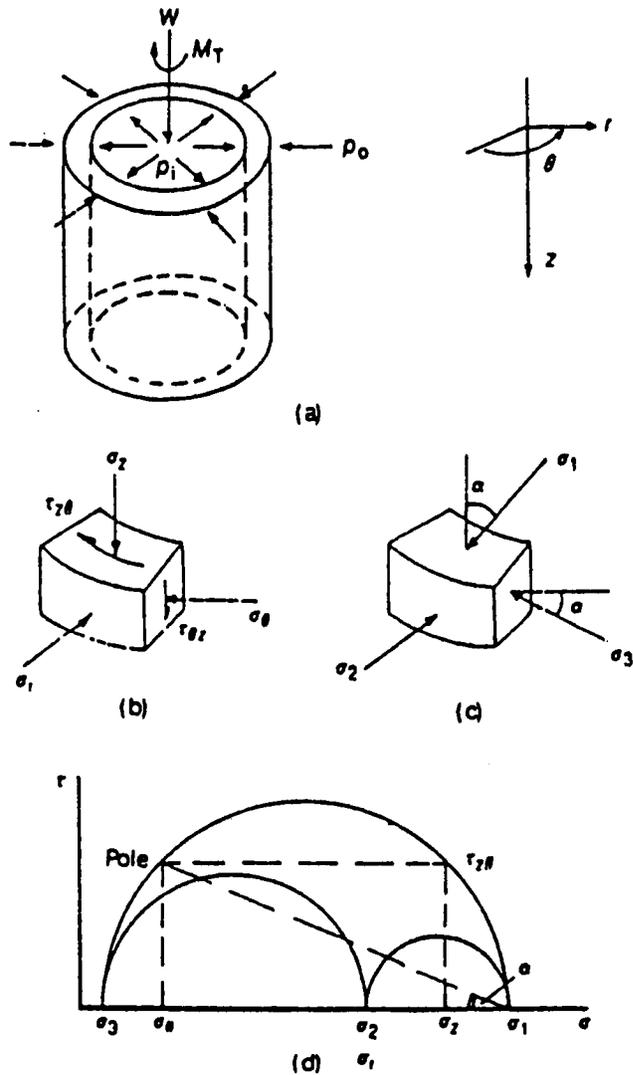
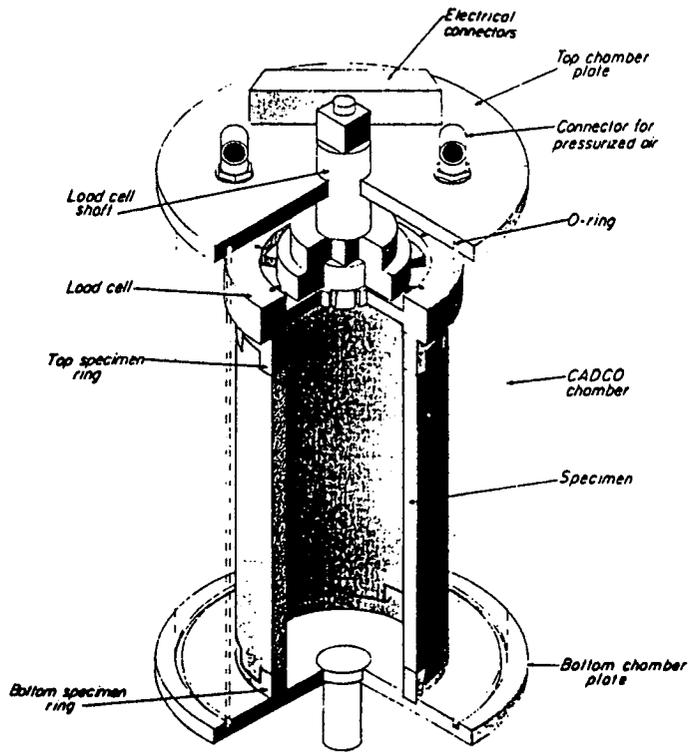
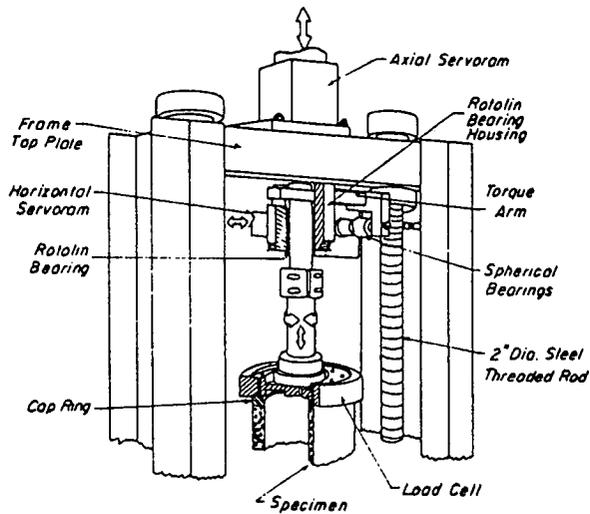


Figure 4.7 Idealized stress conditions in a hollow cylinder test: (a) loading; (b) stresses on wall element; (c) principal stresses on wall element; (d) Mohr circle representation of stresses



a. Axonometric view



b. Dynamic loading system

Figure 4.8 Hollow cylinder test system

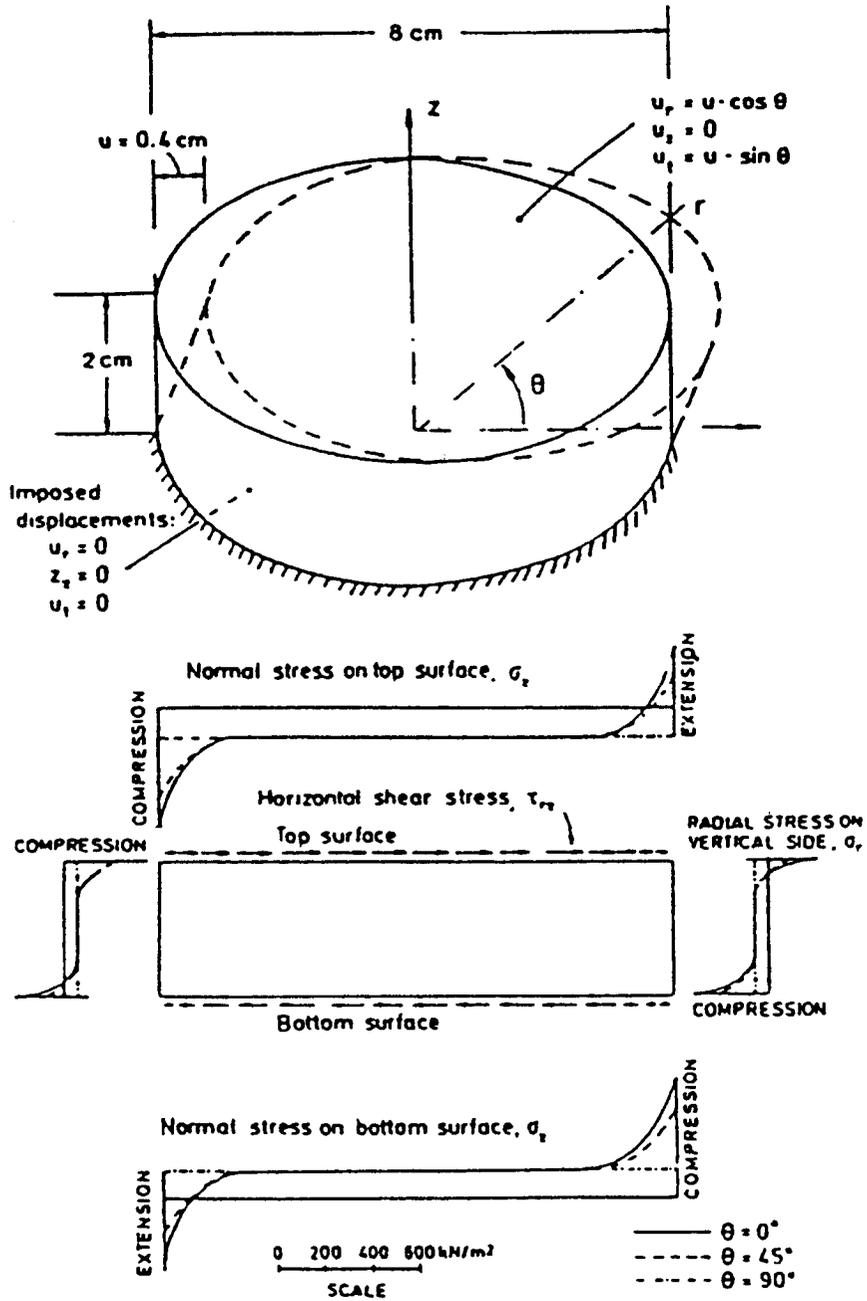


Figure 4.9 Normal stress distribution at specimen boundary  
(After Lucks et al., 1971)

to triaxial testing. The simple shear test can approximate field situations that are characterized by a pure shear stress state or one close to it. It is the simplest test that permits controlled rotation of the principal axes of strain and stress (Figure 4.9).

The simple shear test has not been extensively used for measuring asphalt-concrete properties: however, it appears suitable for investigating the rutting propensity of asphalt concrete because rutting is predominantly caused by plastic shear flow.

The Laboratoire Central des Ponts et Chaussées (LCPC) (Bonnot, 1986) has used a simple shear test to determine fatigue characteristics of asphalt concrete (Figure 4.10). The test is performed under imposed strain amplitudes on almost rectangular specimens (Figure 4.11).

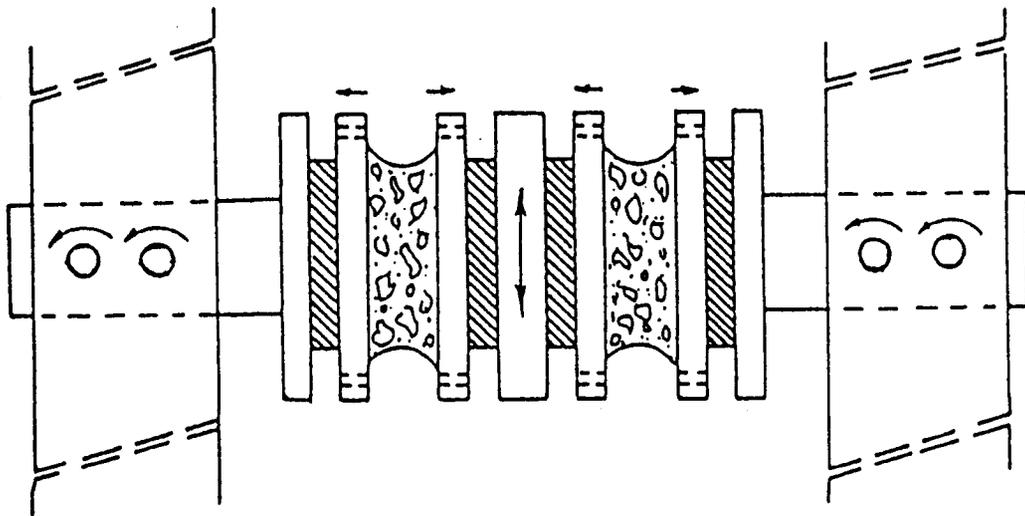
Monismith and Tayebali (1988) employed the simple shear test to compare the response of cored specimens obtained from field pavements with the response of specimens compacted with the California kneading compactor. Briquet specimens, 4 in. in diameter and 2.5 in. thick, were tested in the apparatus depicted schematically in Figure 4.12. Although only creep response was measured, the apparatus is also capable of applying repeated or dynamic loads over a range of frequencies for the determination of resilient shear modulus, dynamic shear modulus, or shear damping response under stress or strain control and with or without stress reversal.

## 4.8 Wheel-Track Tests

All proposed methods for estimating rutting need further field and test-track validation. A complete mechanistic validation should include determining whether the correct plastic strain profile, both with depth and laterally, can be estimated.

Bonnot (1986) has described procedures used by the LCPC for practical mixture design. He has emphasized that, for design applications, laboratory simulation of rutting must duplicate stress conditions in actual pavements. In the LCPC design procedure, a wheel-track test (Figure 4.13) is used for measuring ruts created by the repeated passage of a wheel over prismatic asphalt-concrete specimens. Each specimen is a plate measuring 500 by 150 mm and is 100 mm thick. It is placed in a metal frame and rests on a steel base plate. The specimen can be removed from an actual pavement but generally is compacted in the laboratory with a pneumatic tire.

Rutting is measured by the relative percentage reduction in the thickness of asphalt concrete in the wheel path. The test is terminated after  $10^5$  cycles, unless the rut depth exceeds 15 percent prior to this time.



**Figure 4.10 Schematic representation of loading mechanism for shear testing  
(After Bonnot, 1986)**

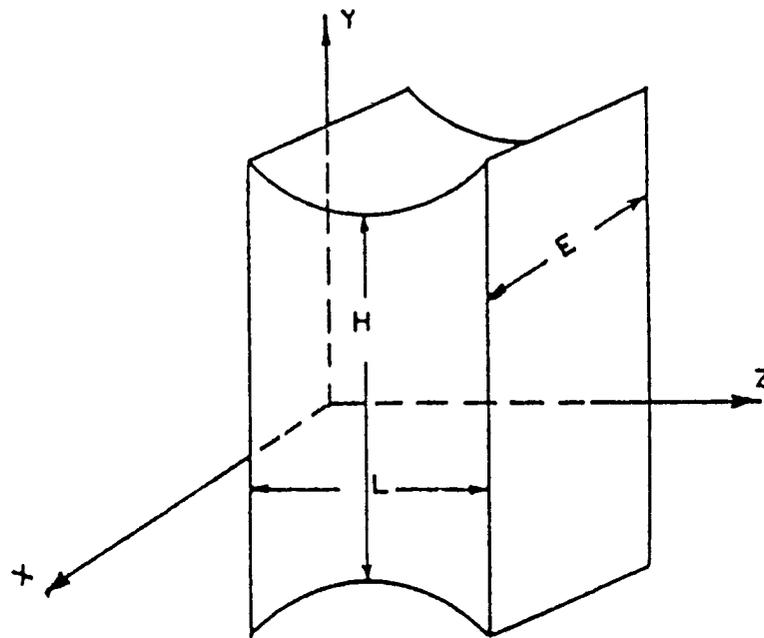


Figure 4.11 Test specimen configuration for shear test (After Bonnot, 1986)

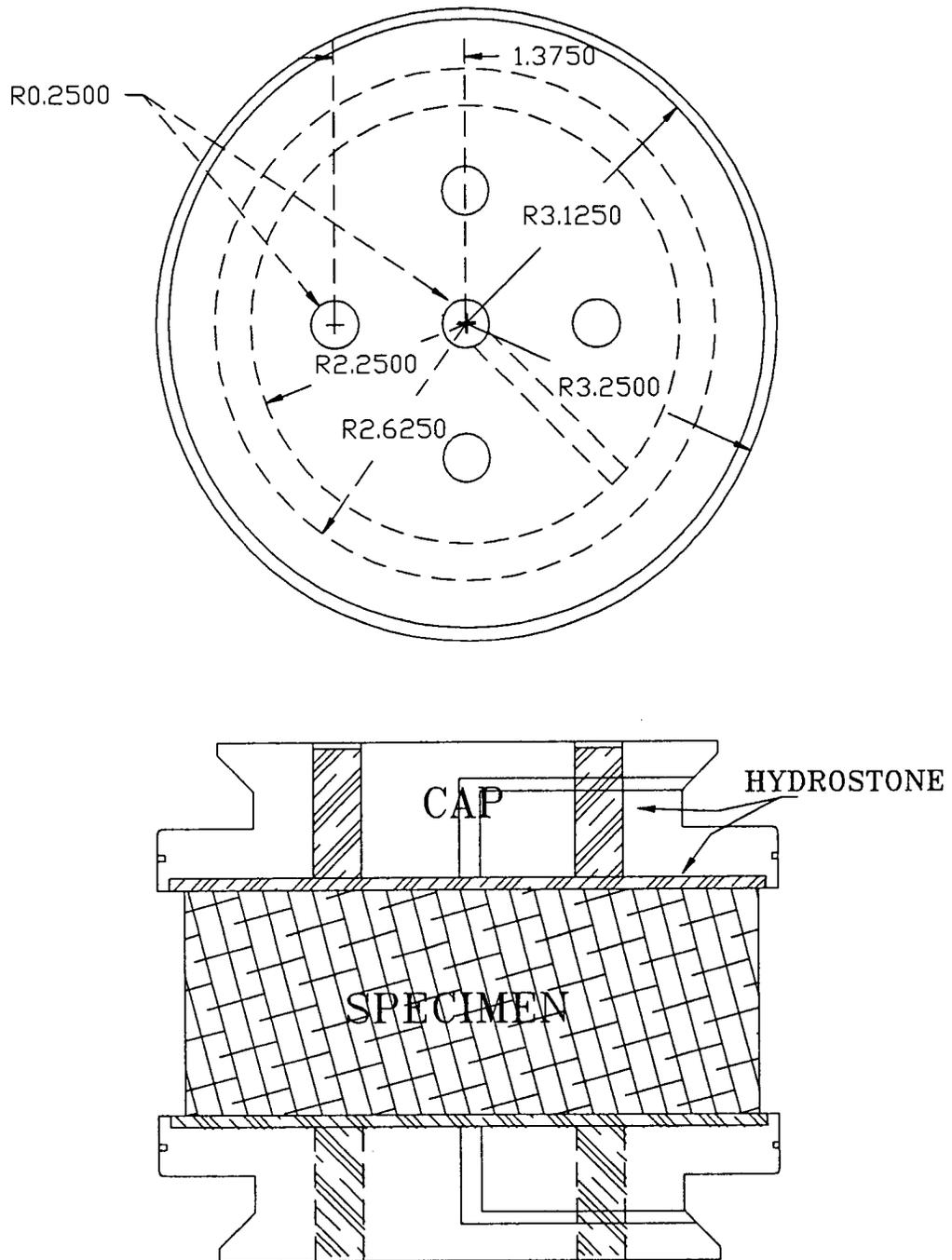


Figure 4.12 Shear apparatus

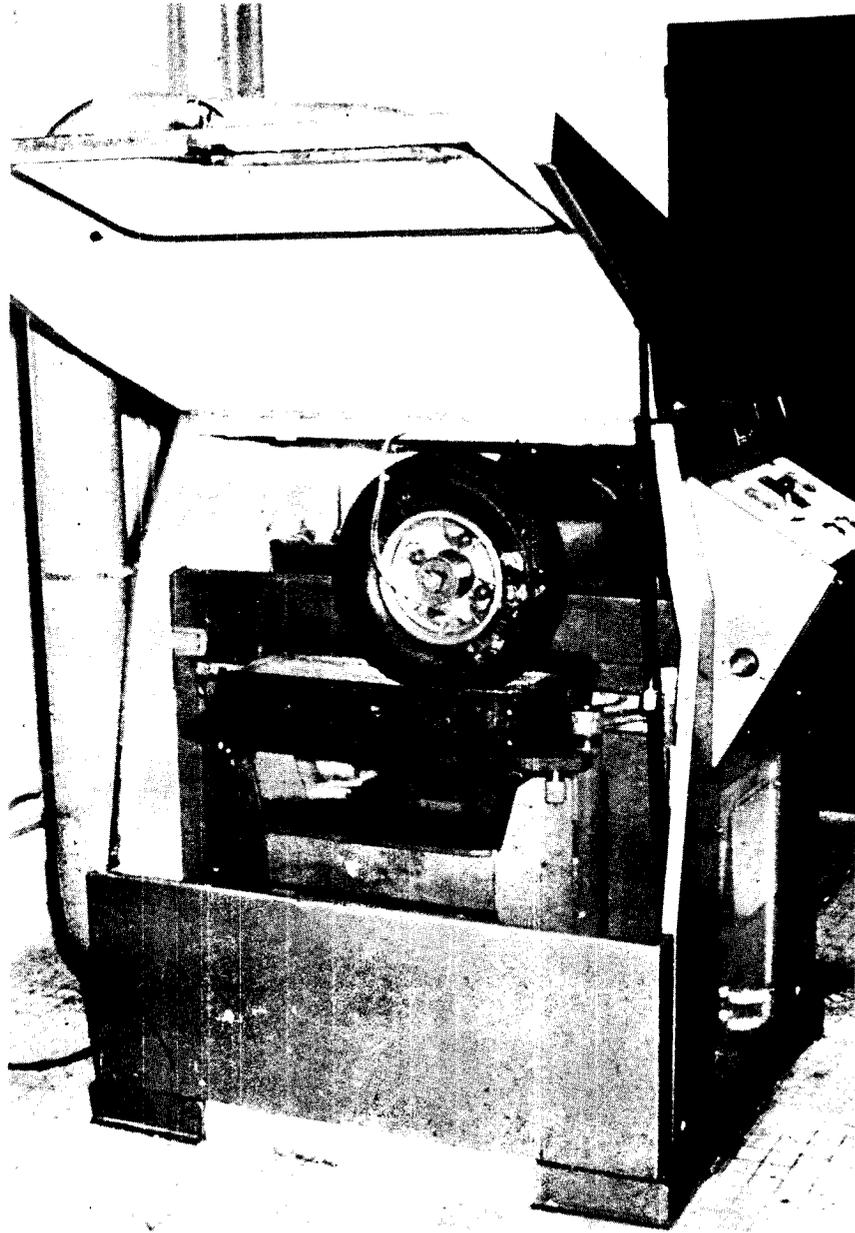
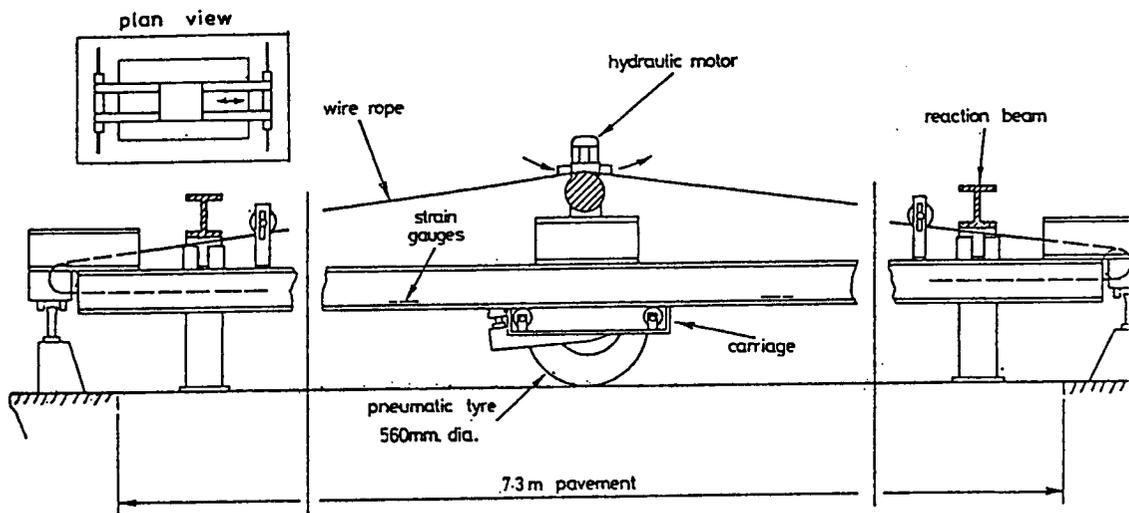


Figure 4.13 LCPC wheel-tracking rutting-test machine (After Bonnot, 1986)

Generally rut depths are measured by stopping the test at 30, 100, 300, 1,000, 3,000, and 100,000 cycles. Tests are executed over the temperature range of 50°C to 60°C to reproduce the most unfavorable pavement conditions expected in France. The tire passes over the center of the specimen twice each second, and the loading time at the center is approximately 0.1 seconds.

The Nottingham Pavement Test Facility (Figure 4.14) allows instrumented pavement sections, 16 ft. long and 8 ft. wide, to be constructed in a 5-ft. deep test pit. Testing is carried out under controlled temperatures with a rolling tire, loaded to a maximum of 2 tons and inflated to 73-psi contact pressure, travelling at speeds up to 10 mph. The facility enables the collection of detailed pavement performance data under carefully controlled conditions.

Full-scale instrumented circular facilities have also been developed in Nantes, France. While circular test tracks may be useful for studying fatigue, there is some concern about their efficacy to study realistically the development of permanent deformation under repeated trafficking. This concern stems in part from the state of stress imposed by the tire in this circular loading path as compared to the stress state which develops from tires following a straight trajectory. Field test sections on existing roads have also been used to study pavement response. The major advantage of studies of this type is that they are representative of the response under real traffic patterns and environment conditions. Their major disadvantage is the confounding nature of uncontrollable elements that may affect the experiment and complicate its interpretation.



**Figure 4.14 Side view of Nottingham pavement testing facility  
(After Brown and Bell, 1979)**

# 5

## Selection of Test Systems

From the information developed and summarized in Section 4.0, test methods were evaluated to determine the extent to which each might be effective as a standard testing procedure for measuring material properties representative of rutting in actual pavements. Special emphasis was placed on the ability of each test to represent in situ stress states and, to a somewhat lesser extent, on its simplicity. The two major criteria were:

### 1) Field Simulation

- A state of stress representative of the shear stresses causing permanent deformation in situ;
- Repeated or dynamic loading (with stress reversal) as a close proxy for in situ traffic loading; and
- Supplemental data, if possible, useful in mechanistic analyses.

### 2) Simplicity

- Ease of specimen fabrication;
- Minimum quantities of material for fabricating specimens in the laboratory;
- Capacity to test cores obtained from existing pavements;
- Compatibility with equipment currently available in materials laboratories; and
- Minimum cost of new equipment or supplemental devices required to adapt existing equipment.

In addition to these two major criteria, consideration was also given to other factors as identified in Table 5.1. The ratings shown in Table 5.1 are by order of preference. Test-track and hollow cylindrical tests were not included in the overall rating because they are considered too complex to be implemented as standard tests. They are, however, considered suitable for research and for the validation of alternate predictive models. Therefore, the shear test remains the only test where shear stresses can be directly applied and where testing sequences with stress reversal can be easily implemented.

The ratings of Table 5.1 represent the subjective judgement of the authors of this report, based on information available in the published literature. The various laboratory test methods will be further evaluated in subsequent phases of Project A-003A.

Table 5.1. Comparison of various test methods for permanent-deformation evaluation.

TEST METHOD	SAMPLE SHAPE	MEASURED CHARACTERISTICS	ADVANTAGES AND LIMITATIONS	FIELD SIMULATION	SIMPLICITY	OVERALL RANKING
Uniaxial static (creep)	Cylindrical 4 in diameter 8 in high	Creep modulus vs time Strain vs time	Wide spread, well known Easy to implement; equipment generally available in labs State of stress contains shear components in the Mohr-Coulomb representation More technical information			
Uniaxial repeated		Resilient modulus Permanent deformation vs cycles Poisson's ratio	Better expresses traffic conditions Equipment is more complex	5	1	3
Uniaxial dynamic		Dynamic modulus Damping ratio Poisson's ratio Permanent deformation vs cycles	Capability of determining the damping as a function of frequency for different temperatures			
Triaxial static (creep confined)	Cylindrical 4 in diameter 8 in high	Creep modulus vs time Strain vs time	More states of stress can be obtained State of stress contains shear components in Mohr-Coulomb representation			
Triaxial repeated		Resilient modulus Permanent deformation vs cycles Poisson's ratio	Better expresses traffic conditions Equipment is more complex Requires a triaxial chamber	4	3-4	2
Triaxial dynamic		Dynamic modulus Damping ratio Poisson's ratio Permanent deformation vs cycles	Capability of determining damping as a function of frequency for different temperatures			
Test tracks	Slabs	Rut profile vs number of passages In depth strain/stress profile	States of stress duplicate field conditions Fundamental material properties cannot be obtained Good method for verification of predictive models Requires special equipment that can be costly	1	6	Not suitable for routine use

Table 5.1. Comparison of various test methods for permanent-deformation evaluation, continued.

Diametral static (creep)			Creep modulus vs time Permanent deformation vs time	Easy to implement Field cores can be easily obtained Shear stress field not uniform			
Diametral repeated	4 in diameter 2.5 in high		Resilient modulus Permanent deformation vs cycles	State of stress is predominantly tension Equipment is relatively simple in static test	6	2	4
Diametral dynamic			Dynamic modulus Damping ratio Permanent deformation vs cycles	For repeated and dynamic tests, the complexity of the equipment is similar to that of triaxial repeated and dynamic equipment			
Hollow cylindrical	1 in wall thickness 18 in high 9 in external diameter		Dynamic axial modulus Dynamic shear modulus Axial damping ratio Shear damping ratio Axial permanent deformation vs cycles Shear permanent deformation vs cycles	Almost all states of stress can be duplicated Capability of determining damping as a function of frequency for different temperatures for shear as well as axial Sample preparation is tedious Expensive equipment Cores cannot be obtained from pavement	2	5	Not suitable for routine use
Simple shear static (creep)			Shear creep modulus vs time Shear permanent deformation vs time	Shear stress can be directly applied to the specimen Cores can be easily obtained from existing pavements			
Simple shear repeated	4 in diameter 2.5 in high		Shear permanent deformation vs cycles Resilient shear modulus	Better expresses traffic conditions	3	3-4	1
Simple shear dynamic			Dynamic shear modulus Shear permanent deformation vs cycles Damping ratio	Capability of determining the damping as a function of frequency for different temperatures Equipment not generally available			

# 6

## Discussion

Based largely on information summarized thus far, the following discusses improvements to testing and analysis systems that would permit the engineer to evaluate rutting problems with a greater degree of confidence.

Trenching studies at the AASHO Road Test (Highway Research Board, 1962) and test-track studies reported by Hofstra and Klomp (1972) indicated that shear deformation rather than densification was the primary rutting mechanism. Similar conclusions can be found in recent work of Eisenmann and Hilmer (1987) who concluded that rutting results primarily from deformation flow at constant volume.

The use of the Heavy Vehicle Simulator (HVS) (Freeme, Maree, and Viljoen, 1982) in the evaluation of pavement behavior has shown that permanent deformation in the asphalt concrete cannot be completely accounted for by the vertical subgrade strain criterion. From the HVS studies, it was also concluded that better characterization of the shear properties of asphalt concrete is necessary if accurate predictions of rutting are to be made.

Because, in general, shear distortion is the most important mechanism causing permanent deformation in a pavement structure, separating the stress state into  $p$  (volumetric) and  $q$  (shear) components is a rational and logical approach for selecting appropriate stress conditions for laboratory testing. Both Brown and Bell (1977) and Freeme (1973) have found that permanent deformation is primarily caused by the shear component,  $q$ : the mean normal stress,  $p$ , was found to contribute insignificantly to the accumulation of permanent strain. Since permanent strain is primarily associated with  $q$ , the variation of  $q$  within the pavement structure, particularly near the surface, should be of great concern in selecting appropriate stress states for laboratory testing.

From theoretical analysis, Brown and Bell (1977) also found that approximately the same amount of rutting was induced for distances up to 4.4 inches from the center of the wheel load, a further reflection of the role of shear stresses in inducing permanent deformation.

For a theoretical method to be mechanistically correct, it must predict the variation of permanent strain with depth in the bituminous layer. For thin sections and small wheel loadings, Hofstra and Klomp (1972) found relatively little variation in permanent strain with depth in their small-scale test section. For thicker layers, the calculated strain as illustrated in Figure 6.1 is much larger in the upper part of the layer than in the bottom part, whereas the permanent deformation was found to be only slightly larger.

Brown and Bell (1977) also measured the variation of permanent strain with depth in the Nottingham Test Track (Figure 6.2). The 4.7-in. bituminous layer rested directly on a silty clay subgrade and was subjected to 100,000 repetitions of a 3,600-lb. load. The measured variation in vertical permanent strain for the bituminous mixture placed at 6-percent voids was parabolic in shape, being somewhat larger at the top and bottom and smaller near the center of the layer (Figure 6.2a). A distinctly different pattern was observed for the high-void mixture (Figure 6.2b).

Using layer-strain analysis, Meyer and Haas (1977) found that rutting in the tensile zone of the bituminous layer was approximately three to four times that in the compression zone. In contrast, Battiato, Ronco, and Verga (1977) calculated a relatively uniform profile using viscoelastic theory (Figure 6.3). Comparison of predicted profiles with measured variations in test tracks indicates that, in many instances, layer-strain theory, as presently applied, estimates permanent strains in the tensile zone which are greater than those that apparently develop in actual pavements. The above considerations stress the apparent discrepancy between actual and theoretically calculated distributions of rutting with depth.

The variation of permanent strain with depth has been measured in test tracks having relatively thick bituminous-concrete surfaces. It seems to be a function of the thickness of the bituminous layers. For example, rutting at the AASHO Road Test was almost directly proportional to the thickness of the bituminous layer up to a certain limit. Beyond that, the average plastic strain in the layer decreased. This suggests that the plastic strain near the bottom of the layer becomes very small for thick bituminous layers. With increasing depth, the resistance to plastic flow is greater due in part to the presence of surrounding material. The variation of plastic strain with depth is also probably influenced by the magnitude of the wheel loading and the number of load repetitions.

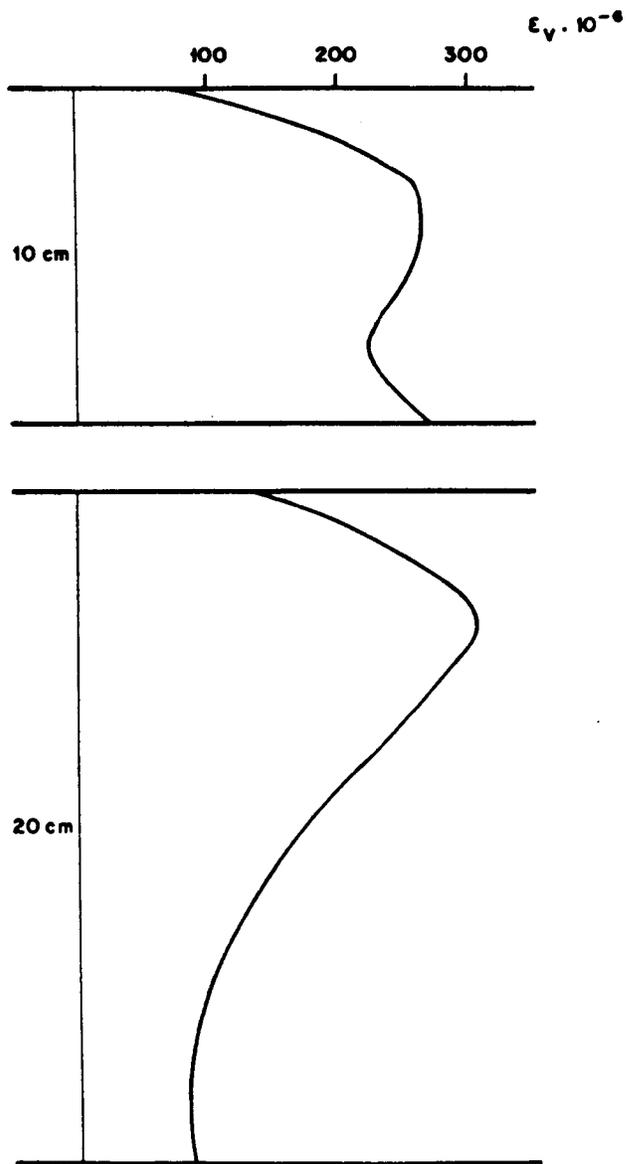
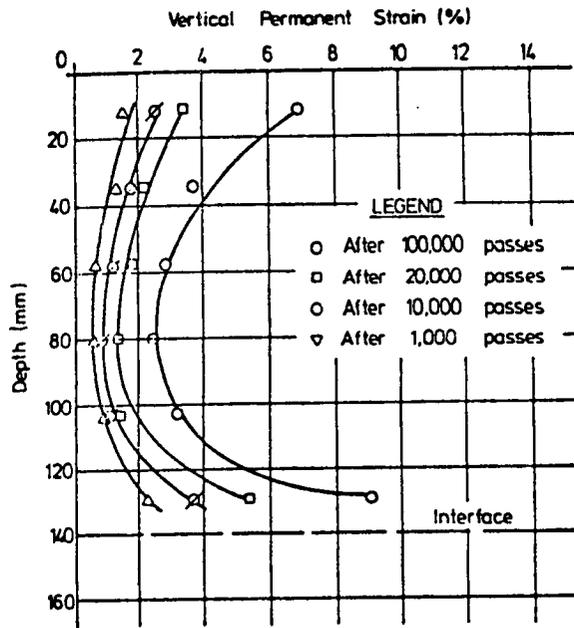
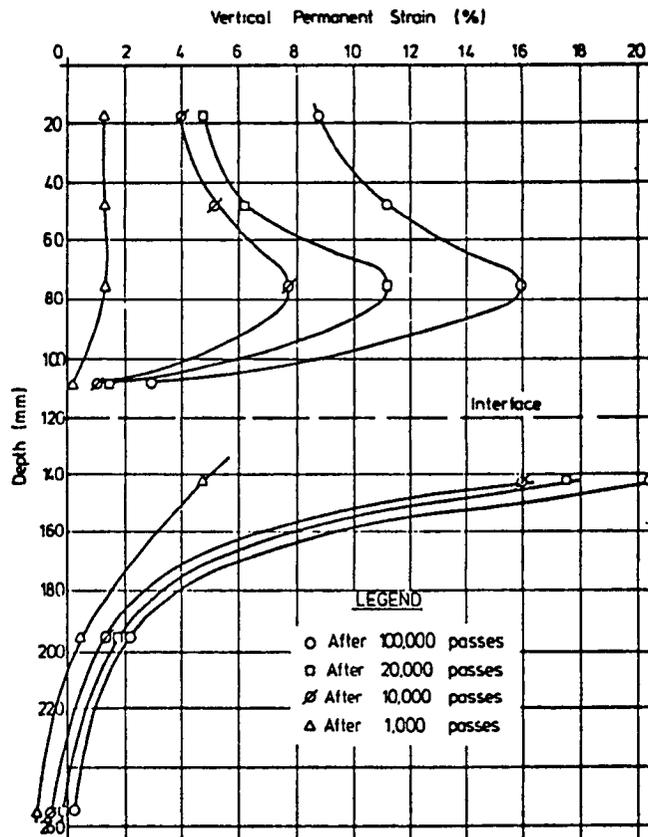


Figure 6.1 Distribution of calculated vertical strain,  $\epsilon_v$ , over layer thickness  
(After Hofstra and Klomp, 1972)

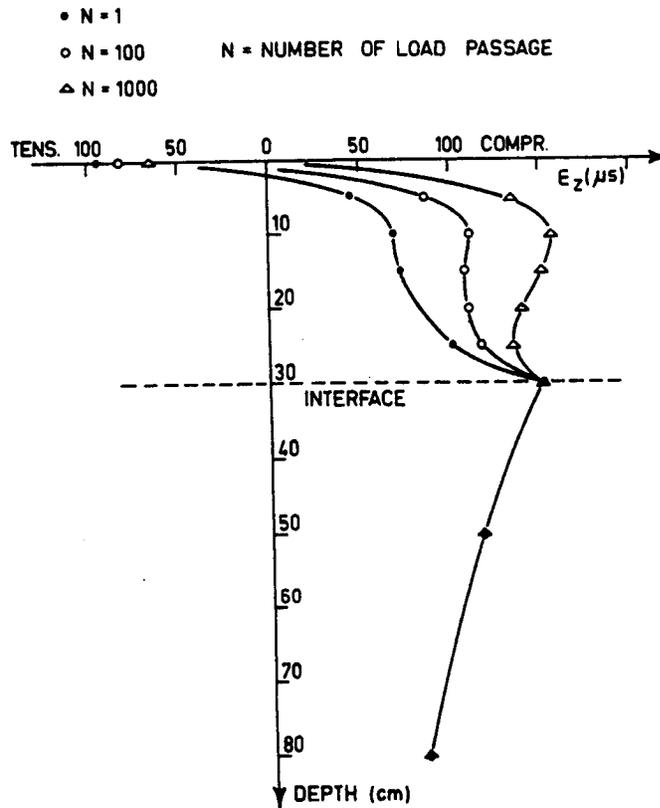


**a. Low-void (6%) mixture**



**b. High-void (10%) mixture**

**Figure 6.2 Measured distribution of permanent strains in dense bitumen macadam (After Brown and Bell, 1977)**



**Figure 6.3 Accumulation of the vertical deformation vs. depth along the vertical of the center of the mass of the load system (After Battiato, Ronco, and Verga, 1977)**

It is necessary at this stage to emphasize that rutting depends on the following two factors:

- *Material Properties.* Each material has its own capacity to withstand permanent deformation. For asphalt mixtures, this depends on aggregate (surface texture, gradation, voids, etc.), binder (type, content, etc.), and additives. By varying one or more of these factors, the capacity of the mixture to resist permanent deformation is altered.
- *States of Stress.* For a given set of materials, representing the several layers of a pavement, the extent of rutting depends upon the states and repetitions of stress that are imposed by traffic loads. Therefore, all factors affecting the states of stress, such as structural characteristics of the pavement (layer thicknesses and moduli) and vehicle characteristics (load, tire distribution, and tire contact pressures), must be considered in rutting analyses.

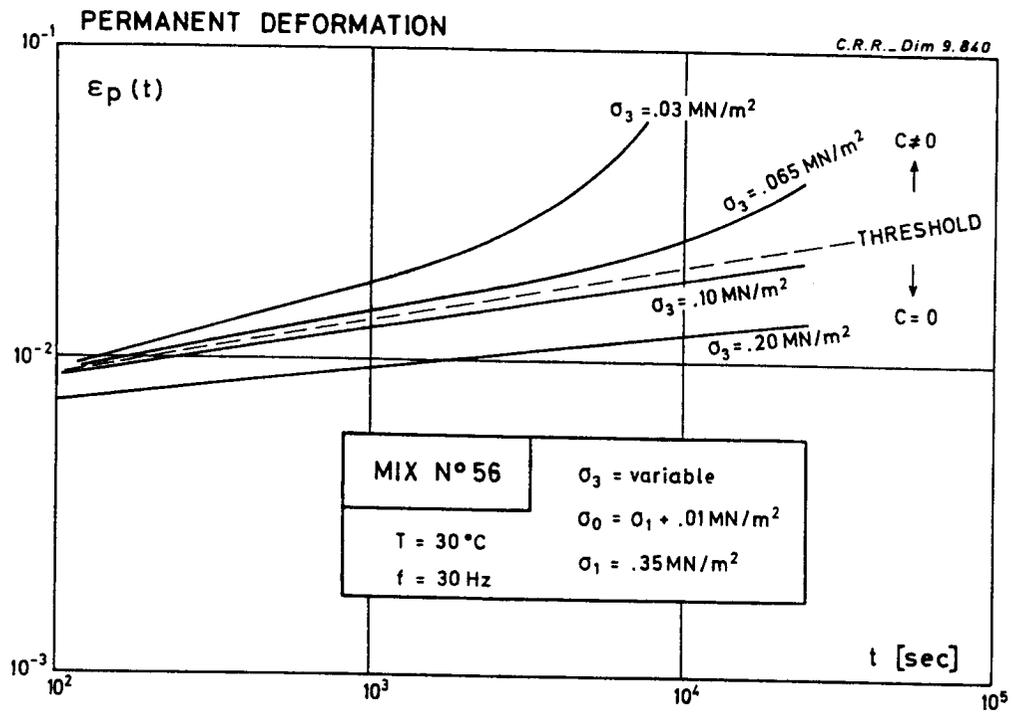
Rutting can be reduced by using materials that better resist permanent deformation and/or by controlling loading conditions either by varying the pavement structure or the vehicular characteristics (e.g., tire pressures) to avoid the critical states of stress that intensify rutting.

Several researchers have developed permanent deformation laws based on state of stress and temperature. For example, Francken (1977) has shown that, for each mixture, there is a state of stress and temperature above which it becomes unstable (i.e, permanent deformation develops at an accelerating rate) (Figure 6.4). Francken also identified a threshold in the stress field above which a given mixture exhibits premature failure (Figure 6.5). From these data, he developed a permanent-deformation law relating axial permanent deformation under creep loading to state of stress, frequency of loading, and temperature (Table 3.1).

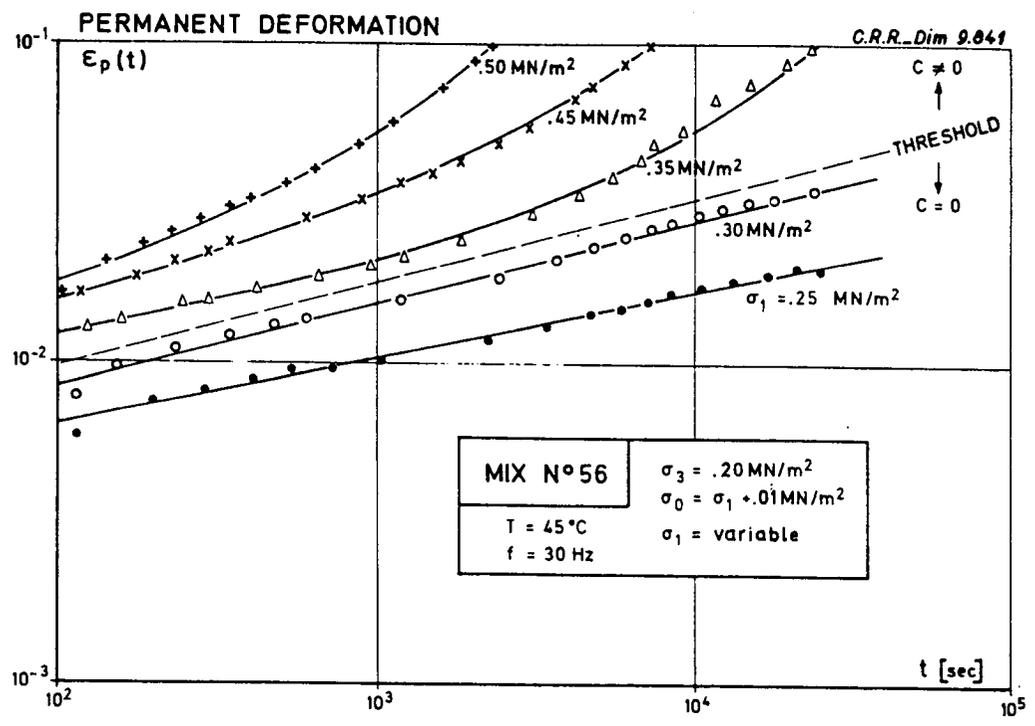
Others have predicted permanent deformation behavior under states of stress encountered beneath the center line of the loads using either uniaxial or triaxial loading. However, Thrower (1978) suggested that the triaxial test is not suitable for investigating materials in the upper layers of asphalt-concrete pavements because it cannot duplicate the states of stress that are encountered near the load.

The following summarizes important findings of the above:

- 1) In densely compacted pavements, shear stresses are largely the cause of permanent deformation in the upper asphalt-bound layers.



**a. Influence of confining stress,  $\sigma_3$**



**b. Influence of axial stress,  $\sigma_1$**

**Figure 6.4 Creep curves (After Francken, 1977)**

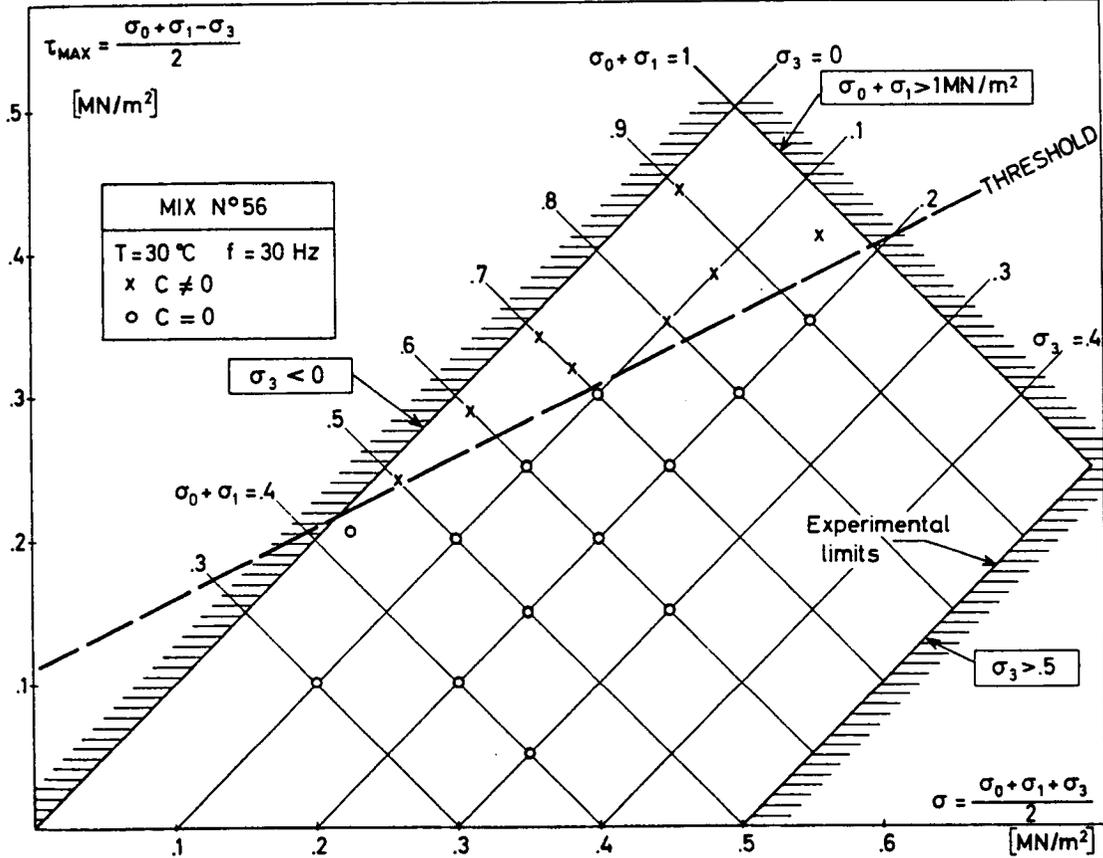


Figure 6.5 Diagram representing the maximum shear stresses occurring in different experiments (shaded area is beyond the range of stresses obtainable with equipment) (After Francken, 1977)

- 2) Current analytical models focus on the rut depth under the center line of loading and do not consider the contribution to rutting of shear stresses at the edges of the tires.
- 3) Analytical models differ greatly in their estimation of the variation of permanent strain with depth beneath the pavement surface.
- 4) Theoretical predictions of the distribution of permanent deformation with depth fail to match test-track and field measurements.

Accordingly, improved methods for analyzing rutting propensity and designing deformation-resistant mixtures for a variety of traffic and environmental conditions are likely to require the following:

- 1) Development of a predictive model that takes into account shear stresses under the edges of the tires.
- 2) Extension of the stress analysis from the centerline of loading to the entire rutting zone using, for example, a finite element technique.
- 3) Extension of the states of stress under which permanent-deformation characteristics of materials are measured in the laboratory to duplicate the states of stress that are encountered within the entire rutting zone, in particular where the shear stresses are relatively greater than normal stresses (this implies that the rutting propensity of mixtures should be investigated using equipment capable of directly applying shear stresses).
- 4) Continued development of a generalized permanent-deformation law for asphalt concrete which can also incorporate states of stress and/or strain and temperature encountered in pavement sections away from the centerline of the loading. The generalized law, is given as follows:

$$\Delta \epsilon_{p_{ij}} = f(\bar{\epsilon}_{p_{ij}}, \sigma_{ij}, \epsilon_{ij}, T, \omega, C) \quad (6.1)$$

where  $\Delta \epsilon_{p_{ij}}$  is the increment in the generalized state of permanent deformation per load cycle;  $\bar{\epsilon}_{p_{ij}}$  is the generalized state of permanent deformation;  $\sigma_{ij}$  is the generalized state of stress;  $\epsilon_{ij}$  is the generalized state of strain,  $T$  is the temperature;  $\omega$  is the frequency or type of loading;  $C$  is a set of mixture constants determined by executing a very limited number of tests; and  $f$  is a function to be determined after extensive testing using several types of mixtures, equipment, states of stress/strain, temperatures, load types and frequencies, and numbers of repetitions.

A generalized law is needed not only to provide input for predictive permanent-deformation models but also to complement mixture design procedures. If such a function is determined after extensive testing, it is

reasonable to expect that the set of constants that uniquely identify an asphalt-concrete mixture could be determined by a relatively few tests at selected temperatures and states of stress.

- 5) Conduct a series of full-scale field rutting tests for a variety of pavement structures and different traffic loadings (e.g., tire pressures). In these tests, particular attention should be given to measuring states of stress and strain in the asphalt-concrete layer. Strain-measuring rosettes should be used near the pavement surface to measure the variation of permanent shear deformation.

# 7

## Conclusions And Recommendations

This summary report deals primarily with the ability of asphalt-aggregate mixtures to sustain imposed loads without permanent deformation. It considers the basic factors that affect permanent deformation, laboratory tests for measuring mixture resistance to permanent deformation, and methods for analyzing permanent deformation within the upper layer(s) of the pavement structure. The following conclusions and recommendations are based on the information presented herein.

### 7.1 Conclusions

- 1) Rutting in pavements develops gradually with increasing wheel load applications and usually appears as a longitudinal depression in the wheel path with small shoulders to the sides. Rutting is caused by a combination of densification (decrease in volume and, hence, increase in density) and shear deformation: however, shear deformation rather than densification is considered to be the primary cause of rutting in properly constructed pavements.
- 2) Mixture characteristics and test/field conditions that have important effects on rutting of asphalt-concrete pavements include the following:
  - Dense aggregate gradations are desirable to mitigate the effects of rutting in asphalt-concrete layers. When properly compacted, a mixture with a dense and continuous aggregate gradation has fewer voids and more contact points between aggregate particles than open or gap-graded mixtures.

- The texture of the aggregate is important and a rough texture is required, particularly in thicker asphalt-bound layers in hotter climates.
- Mixtures made from angular aggregates (obtained by crushing) are more stable than mixtures made from rounded aggregates.
- Lower viscosity asphalts make the mixture less stiff and, therefore, more susceptible to rutting; harder (more viscous) asphalts should be used in thicker pavements in hotter climates.
- The binder content influences the mixture's ability to resist permanent deformation. Larger binder contents increase rutting potential.
- Laboratory specimens must be compacted to densities comparable to those reached in situ as a result of both construction and subsequent traffic loading.
- When compacted in the field with heavy rollers, mixtures of relatively poor workability are effective in minimizing rutting propensity (to an extent, workability derives from other factors including rough, angular, and densely graded aggregate, stiff asphalt, and low asphalt content.) Such mixtures have a stable arrangement of the mineral skeleton and, thereby, large internal friction.
- Degree of compaction is the primary quality parameter of the placed asphalt mixture, especially when the mixture is critically designed (when it has a low asphalt content to produce high resistance to permanent deformation).
- Temperature has a significant effect on rutting and, therefore, temperatures employed for design are relatively high to reproduce the most unfavorable pavement conditions.
- Changes in the distribution of traffic, especially larger proportions of heavy trucks, may increase the rate of rutting even if the pavement was originally well designed and constructed.
- Larger wheel loads and larger tire inflation pressures increase the rate of pavement rutting.

3) Both layer-strain and viscoelastic methods are presently used for predicting permanent deformations in pavements.

- The layer-strain method is considered a simplified engineering theory for predicting rut depth which allows use of either linear or nonlinear elastic theory together with the permanent-deformation characteristics of asphalt mixtures.

- Nonlinear viscoelastic models are more realistic than linear viscoelastic models for predicting permanent deformations but, because of mathematical complexities and difficulty in defining an appropriate material model, the linear model is most frequently used.
  - Although the viscoelastic method is theoretically sounder than the layer-strain approach, it is more complex and has not been shown to be more accurate in estimating permanent deformations observed in pavements in situ.
  - Common to these two approaches is 1) the determination of the state of stress in the pavement structure and 2) measurement of permanent-deformation characteristics of the materials comprising the pavement section.
  - The use of stress invariants is appropriate for representing the correct stress state for purposes of materials characterization. Their use is particularly advantageous in the tension zone at the bottom of asphalt layers and also for predicting permanent deformations away from the axis of loading. Because shear distortion is the most important mechanism causing permanent deformation in pavements, separating the stress state into volumetric ( $p$ ) and shear ( $q$ ) components is a rational approach for selecting appropriate stress conditions for testing purposes.
  - Given the complexity of the states of stress and the large number of parameters involved in the analysis, computer programs are required for investigating the response of multilayered systems. Linear elastic, nonlinear elastic, and linear viscoelastic models have been implemented in software packages.
  - Finite element methodology has been used in the determination of the state of stresses in pavements. This technique has the capability to better define the propensity for rutting in the entire portion of the pavement structure where rutting can occur.
- 4) Procedures for rutting prediction require that suitable techniques be developed not only for calculating the response of the pavement to load but also for realistically characterizing material properties. The overall objective of materials testing should be to reproduce as closely as practical in situ pavement conditions including the general stress state, temperature, moisture, and general conditions of the affected materials.
- A number of models representing the permanent deformation behavior of asphalt mixtures have been developed. They can be

grouped into three general categories; empirical regression equations; "typical" plastic strain laws; and models based directly on results of laboratory tests. In all cases, laboratory tests are necessary to determine parameters representative of permanent-deformation behavior.

- Compaction is critical in preparing samples for laboratory evaluation. The purpose of laboratory compaction is to simulate, as closely as possible, the actual compaction produced in the field. Factors such as the orientation and interlocking of aggregate particles, the extent of interparticle contact, air-void content and structure, and number of interconnected voids should be closely reproduced.
  - Tests presently used to characterize the permanent-deformation behavior of pavement materials are of the following general types: uniaxial, triaxial, diametral, and wheel-track tests. These tests have been designed to evaluate the elastic, viscoelastic, plastic, and strength parameters of asphalt mixtures.
- 5) All proposed methods for estimating rutting require field and test-track validation. A complete mechanistic validation must assure that the profile of permanent strain can be accurately estimated. Test sections on existing roads have been used to study pavement response: they are advantageous because they capture the effects of real traffic patterns and real environmental conditions.
  - 6) Comparisons of predicted permanent-strain profiles with measurements in test tracks indicate that, in many instances, layer-strain theory overestimates permanent strains in tensile zones. Test-track results suggest that the permanent strain is quite small near the bottom of thick bituminous layers. The above suggests apparent discrepancy between the actual distribution of permanent strain with depth and the theoretically calculated distribution.
  - 7) Although many researchers recognize that shear stress is the main mechanism causing rutting, suitable laboratory test methods and theoretical models are not yet available for properly treating shear-induced permanent deformation.
  - 8) Current analytical models predict only the rut depth under the center line of loading and do not consider the contribution to rutting caused by the development of shear stresses at the edges of the tires.

## 7.2 Recommendations

The above conclusions suggest that significant new developments are required before sufficiently reliable test procedures, analytical models, and design systems are available. They provide the basis for development of a theoretically sound analysis procedure and related test methodology which will permit more reliable mixture designs and more appropriate modeling of the real conditions under which rutting occurs. Thus the following recommendations are offered:

- 1) A predictive model should be developed that directly takes into account the shear stresses developed within the entire zone of permanent deformation, extending outward from the centerline of loading at least to the tire edges. This can be achieved using a finite element technique.
- 2) The states of stress under which permanent-deformation characteristics of materials are obtained in the laboratory should be extended. Laboratory tests should duplicate the states of stress that are encountered within the entire rutting zone, in particular where the shear stress is relatively greater than the normal stress. Accordingly, the rutting propensity of mixtures should be measured using equipment capable of directly applying shear stresses.
- 3) The development of a generalized permanent-deformation law for asphalt concrete should continue. This law should consider states of stress and/or strain and temperatures encountered in pavement sections away from the centerline of loading. The law should have the functional structure of Equation 6.1 and should be designed not only to provide input for permanent deformation modeling but also to complement mixture design procedures. After extensive testing, it is reasonable to expect that a set of constants that uniquely identify an asphalt-concrete mixture could be determined by just a few tests at selected temperatures and states of stress making the approach attractive for conventional design practice.
- 4) To validate the test methods and analytical system, full-scale field tests should be conducted on pavements composed of different structures and subjected to different patterns and characteristics of traffic (e.g., tire pressures). Emphasis should be placed on measuring the states of stress and strain in the asphalt-concrete layer including shear deformations.

# Appendix A

## Hypotheses and Recommended Test Program

### A.1 Hypotheses

Rutting (permanent deformation) in an asphalt-concrete layer is caused by a combination of densification (volume change) and shear deformation, each resulting from the repetitive application of traffic loads. For properly constructed pavements, shear deformations, caused primarily by large shear stresses in the upper portions of the asphalt-aggregate layer(s), are dominant.

Pavement rutting cannot be estimated with sufficient accuracy and reliability using current mechanistic procedures which are based on either 1) linear viscoelastic models or 2) layer-strain algorithms<sup>10</sup>. However, finite element techniques are now available that are well adapted to the analysis of permanent deformation in pavement structures. They can consider the entire rutting zone, including that near the tire walls, and can effectively handle complex constitutive relationships as well as the transverse distribution (wander) of traffic. For the analysis of surface rutting, supporting layers can be represented simplistically, for example, as "equivalent" linear elastic foundations.

The permanent deformation response of asphalt-aggregate mixtures to loading can be characterized by a generalized permanent-deformation law as follows:

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<sup>10</sup>A linear (or nonlinear) elastic model is used to estimate the state of stress in sublayers of the asphalt-bound layer(s), and laboratory testing provides the estimate of permanent deformation corresponding to that stress state. Permanent deformation at the pavement surface is estimated by the summation of the deformations in the sublayers.

$$\Delta \epsilon_{pij} = f(\bar{\epsilon}_{pij}, \sigma_{ij}, \epsilon_{ij}, T, \omega, C) \quad (A.1)$$

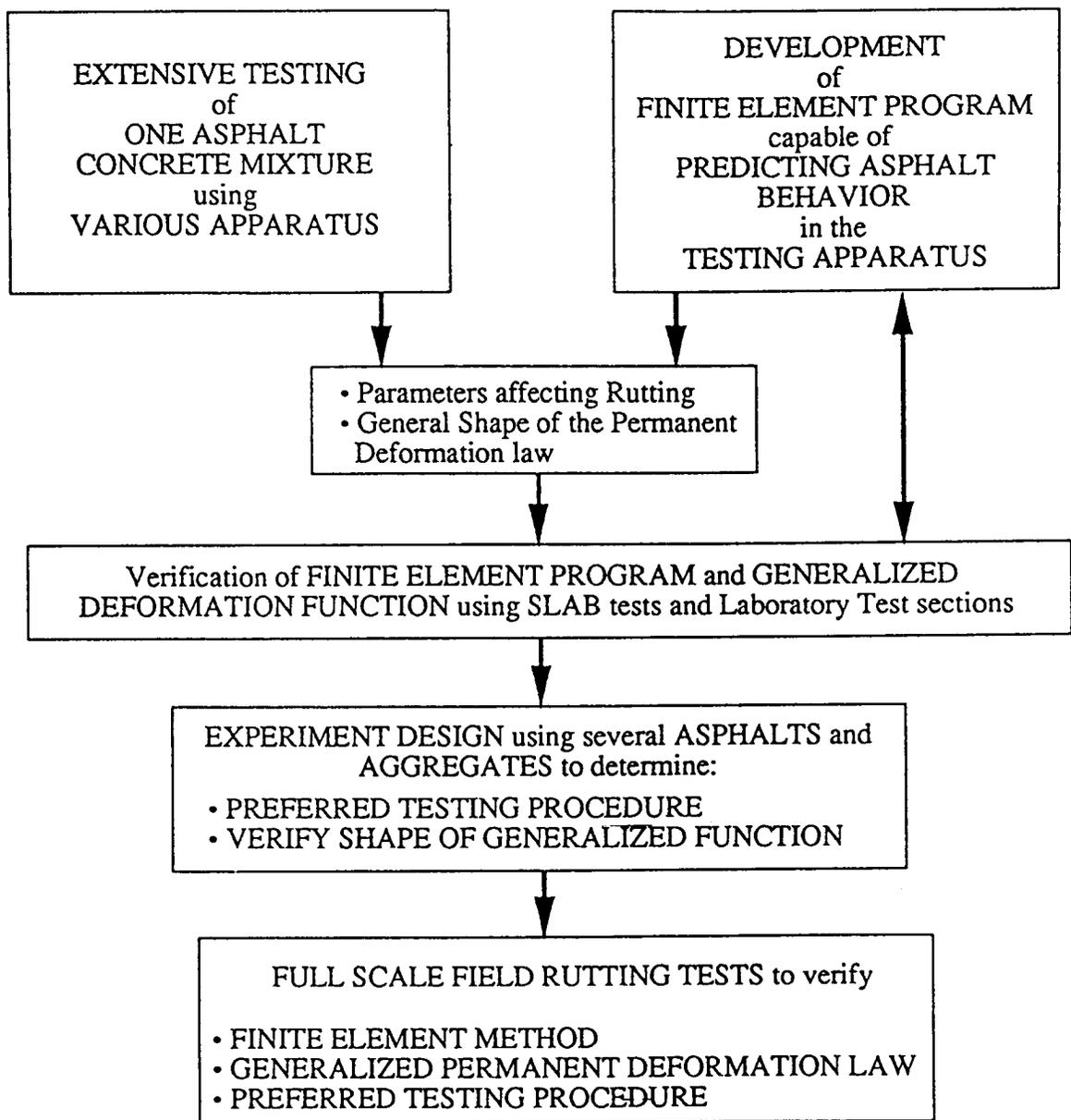
where  $\Delta \epsilon_{pij}$  is the increment in the generalized state of permanent deformation per load cycle;  $\bar{\epsilon}_{pij}$  is the generalized state of permanent deformation;  $\sigma_{ij}$  is the generalized state of stress;  $\epsilon_{ij}$  is the generalized state of strain;  $T$  is the temperature;  $\omega$  is the frequency (or time) and type of loading;  $C$  is a set of mixture constants determined by executing a very limited number of tests; and  $f$  is a function to be determined after extensive testing using several types of mixtures, equipment, states of stress/strain, temperatures, load types and frequencies, and numbers of repetitions.

Repetitive loading in shear is required in order to accurately measure, in the laboratory, the influence of mixture composition on resistance to permanent deformation. Laboratory tests must closely duplicate the states of stress that are encountered within the entire rutting zone, particularly near the pavement surface and off the loading centerline where shear stresses are relatively greater than normal stresses. Specifically, it is the shear stress and/or strain invariants that dictate mixture response in shear. Because of the accelerating rate at which permanent deformation accumulates at higher temperatures, laboratory testing must be conducted at temperatures simulating the highest levels expected in the paving mixture in service. Furthermore, both laboratory testing and the deformation law must recognize that the accumulation of permanent strain in asphalt mixtures is a nonlinear function of the number of loading cycles and, as critical levels of strain are reached, permanent strains increase rapidly.

## A.2 General Approach and Test Development

One of the primary objectives of the permanent deformation study is to develop a generalized permanent-deformation law for asphalt concrete which can incorporate states of stress and/or strain encountered in pavement sections away from the centerline of loading. The generalized law, depicted by Equation A.1, is needed not only to provide input for predictive permanent-deformation models but also to complement mixture design procedures. If such a function can be determined after extensive testing, it is reasonable to expect that the set of constants uniquely characterizing an asphalt-concrete mixture can be determined by a relatively few tests at selected temperatures and states of stress. The combined experimental-analytical plan for developing and validating a generalized permanent-deformation law includes the following steps (see Figure A.1):

- 1) Execute laboratory tests in several apparatus, with known states of stress, to determine material response;



**Figure A.1 Working concept for developing test for evaluation of permanent deformation.**

- 2) Develop a finite element idealization, in FEAP, capable of modeling the behavior of asphalt-aggregate mixtures in the laboratory tests; and
- 3) Verify if the finite element analysis is capable of predicting the response of the mixtures under more complex states of stress.

Initially all tests will be conducted on one mixture (Watsonville granite with the California Valley asphalt at the optimum asphalt content and  $4.0 \pm 0.5$  percent voids) and at one temperature ( $40^{\circ}\text{C}$ ). Other conditions will be investigated when the initial validation is complete. All specimens will be fabricated with the rolling-wheel compactor.

The complete process of testing and validation is described as follows:

- 1) Execute testing under creep, repetitive, and ramp loading according to the test schedule in Table A.1.
- 2) From these tests, identify a limited number of parameters that can characterize mixture behavior. Mechanical behavior is expected to be similar to that of viscoplastic materials with a potential plastic function and an energy function dependent upon the strain level. It is expected that a maximum of five or six parameters will be necessary to characterize mixture response with sufficient accuracy.
- 3) Again testing the "standard" mixture at  $40^{\circ}\text{C}$ , execute a series of "poke" tests to validate the permanent-deformation testing and analysis system. In the poke test, specimens, 14 in. in diameter and 9 in. in thickness, will be loaded axially by means of a circular punch of 4-in. diameter. Although the boundary conditions have yet to be finalized, in all likelihood the cylindrical specimens will be fully restrained at all surfaces except the upper loading surface. Four specimens will be tested, two under creep loading (two load levels) and two under repeated loading (also two load levels).
- 4) Develop a finite-element idealization of the poke test and determine the extent to which measured deformations can be estimated by the model.
- 5) Determine the minimum set of tests necessary to measure mixture parameters needed in the model.

**Table A.1. Test schedule for developing permanent-deformation law**

<b>Configuration</b>	<b>Creep Loading</b>	<b>Repetitive Loading</b>	<b>Ramp Loading</b>
<b>Isotropic</b>	<ul style="list-style-type: none"> <li>• 1 stress</li> </ul>	<ul style="list-style-type: none"> <li>• 1 stress</li> </ul>	<ul style="list-style-type: none"> <li>• 1 stress</li> <li>• 3 rates</li> </ul>
<b>Axial Compression</b>	<ul style="list-style-type: none"> <li>• 3 axial stresses</li> <li>• 3 confining stresses</li> </ul>	<ul style="list-style-type: none"> <li>• 3 axial stresses</li> <li>• 3 confining stresses</li> </ul>	<ul style="list-style-type: none"> <li>• 1 stress</li> <li>• 3 rates</li> </ul>
<b>Axial Tension</b>	<ul style="list-style-type: none"> <li>• 3 axial stresses</li> <li>• 1 confining stress</li> </ul>	<ul style="list-style-type: none"> <li>• 3 axial stresses</li> <li>• 1 confining stress</li> </ul>	<ul style="list-style-type: none"> <li>• 1 stress</li> <li>• 3 rates</li> </ul>
<b>Shear</b>	<ul style="list-style-type: none"> <li>• 3 shear stresses</li> <li>• 3 axial stresses</li> <li>• 3 confining stresses</li> </ul>	<ul style="list-style-type: none"> <li>• 3 shear stresses</li> <li>• 3 axial stresses</li> <li>• 3 confining stresses</li> </ul>	
<b>Hollow Cylinder</b>	<ul style="list-style-type: none"> <li>• 2 axial stresses</li> <li>• 2 shear stresses</li> <li>• 1 confining stress</li> </ul>	<ul style="list-style-type: none"> <li>• 2 axial stresses</li> <li>• 2 shear stresses</li> <li>• 1 confining stress</li> </ul>	
<b>Diametral</b>	<ul style="list-style-type: none"> <li>• 3 axial stresses</li> </ul>	<ul style="list-style-type: none"> <li>• 3 axial stresses</li> </ul>	

- 6) Using those tests, characterize the parameters of a second mixture, similar to the first except for the binder (Boscan will replace California Valley asphalt).
- 7) Perform a repeated-load poke test on the new mixture.
- 8) Model this test with the newly developed code and verify that the deformations can be matched.

Assuming this approach is successful, remaining effort will concentrate on determining the type of apparatus, states of stress, and range in temperatures necessary to characterize the permanent-deformation behavior of other asphalt-aggregate mixtures.

### A.3 Test Program

The test program is designed to compare the mixture testing and analysis (FEAP) system developed through the aforescribed process with the VESYS procedure, an alternative system that incorporates a unique test series and utilizes linear viscoelastic modeling. The results of wheel-track tests will help to evaluate these two systems. Should neither the FEAP nor the VESYS system prove to be acceptable, a primary consideration in evaluating alternate test systems will be their abilities to measure fundamental properties necessary for layer-strain analysis.

**A.3.1 Tests.** The specific test systems to be evaluated will depend on the developments described above. In order to illustrate the testing planned in the next stage, however, it is assumed that repeated-load testing in both axial compression and shear will be required. The following test systems would then be evaluated further:

Agency	Test
University of California	<ul style="list-style-type: none"> <li>• Axial compressive repeated load</li> <li>• Shear repeated load</li> </ul>
University of Nottingham (SWK)	<ul style="list-style-type: none"> <li>• Wheel-track test (slab specimens measuring 305 mm by 305 mm by 50 mm in thickness)</li> </ul>
North Carolina State University	<ul style="list-style-type: none"> <li>• Axial creep (VESYS procedure)</li> </ul>

Each laboratory will prepare its own test specimens.

**A.3.2 Variables Considered.** Table A.2 summarizes the significant variables for this study. For seven, two-level variables, a complete factorial would consist of  $2^7$  or 128 treatment combinations. A  $1/2$  fraction, i.e., 64 treatments, is the smallest that permits estimation of all two-factor interactions. (A  $1/4$  fraction in 32 runs estimates only 15 of the 21 two-way interactions). The  $1/2$  fraction has 35 degrees of freedom associated with higher order interactions. This should be more than enough to adequately estimate testing error without any deliberate replication. It is also possible to split this design into eight blocks of eight runs each, without any loss of information on main effects or two-factor interactions.

**Table A.2. Significant mixture and test variables  
for permanent-deformation study**

Variable	Level of Treatment			Number of Levels
	1	2	3	
Aggregate				
• Stripping Potential	Low		High	(2)
• Gradation		Medium	Coarse	(2)
Asphalt				
• Temperature Susceptibility	Low		High	(2)
• Grade		Medium		
• Content		Optimum	High	(2)
Compaction				
• Air Voids	4 ± %		8 ± %	(2)
Test Conditions				
• Temperature		40°C	60°C	(2) <sup>a</sup>
• Stress	Low		High	(2) <sup>b</sup>
				2 <sup>7</sup>

<sup>a</sup>Only one level (40°C) for wheel-track tests at University of Nottingham (2<sup>6</sup>).

<sup>b</sup>Contact pressures of approximately 80 and 120 psi for wheel-track tests at University of Nottingham.

**Table A.3. Number of samples for permanent-deformation factorial design**

Complete Factorial	$2^7 = 128$
1/2 Factorial	64
Total Number of Samples	320
Estimated Time for Testing	6-9 months

Laboratory/Test	1/2 Fractional = Total
University of California	
• Axial Compressive Repeated Load <sup>a</sup>	64
• Shear Repeated Load <sup>a</sup>	64
North Carolina State University	
• Axial Creep (VESYS Procedure)	64
University of Nottingham (SWK)	
• Wheel-Track Test	128
<b>Grand Total</b>	<b>320</b>

<sup>a</sup>Subject to change following initial test system development and evaluation.

Table A.3 gives the distribution of the numbers of samples for the various test methods at the several laboratories. This assumes that all participating laboratories except the University of Nottingham (SWK) will be investigating seven variables at two levels each. The University of Nottingham's wheel track uses a constant wheel load, and tests will be restricted to just one temperature level. This reduces its tests to a full  $2^6$  factorial for which full replication would be advisable to achieve sufficient precision.

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