

Chapter 7

Methods of Stability Analysis

N. R. Morgenstern and Dwight A. Sangrey

The functional design of a slope requires that the deformations of the earth mass constituting the slope be limited to amounts governed by the use of the ground on or adjacent to the slope. If there is no particular development near the slope, the allowable deformations can be large, provided that the earth mass does not fail with resulting uncontrolled deformations. On the other hand, if there are structures close to the slope or services buried within the soil beneath the slope, smaller deformations may result in unacceptable performance, and functional design would require that the average stress level in the earth mass be lower in this case than in the case in which no development is near the slope.

Analysis is an important component of design, which also includes other equally important components, such as drainage considerations and construction control. In analysis, the mechanical properties of the earth materials are evaluated quantitatively to arrive at a configuration consistent with the performance requirements of the slope. In this chapter, the relative roles of limit equilibrium and deformation analyses are discussed. The principles of limit equilibrium analysis are outlined, and details of the techniques commonly used in practice are given with examples. Both soil and rock slopes are discussed. The chapter concludes with a brief discussion of deformation analysis as it is currently used. Other aspects of design are discussed in Chapters 8 and 9.

ROLES OF LIMIT EQUILIBRIUM AND DEFORMATION ANALYSES

Although the performance of a slope often is dictated by allowable deformations, quantitative prediction of displacements of the slope is seldom undertaken routinely. Instead, analyses are made by the use of limit equilibrium methods, and the performance of a slope is evaluated in terms of its factor of safety (F). There are several reasons for this.

For realistic configurations commonly encountered in design, particularly in the design of cuts and natural slopes,

a method of analysis that predicts deformations to an acceptable degree of accuracy would likely have to include

1. Representative stress-strain relations, including behavior from peak to residual shear strengths;
2. Anisotropy;
3. Variable pore pressure distributions;
4. Nonhomogeneity arising from variation of material properties with depth, layering, and discontinuities;
5. Influence of initial stress; and
6. Construction sequence.

Most of these factors have a major influence on the behavior of a slope; and, even if proven analytical techniques were readily available to predict deformations, the determination of all of these factors in a manner suitable for a deformation analysis would seldom be a practical undertaking. Appropriate analytical techniques are not yet at hand, although impressive progress is being made by means of finite element methods.

The situation is not so pessimistic with regard to fills. Representative stress-strain relations within the working range can be specified with a greater degree of reliability for embankments than for cuts or natural slopes. Moreover, since the soil mass is a processed material, it is generally more uniform and homogeneous than naturally occurring deposits. Hence, deformation analysis has a role in the design of embankment slopes, and that role will be reviewed in greater detail later in this chapter.

The stabilities of natural slopes, cut slopes, and fill slopes are commonly analyzed by limit equilibrium methods. These methods take into account all of the major factors that influence the shearing resistance of a soil or rock mass; this is one of their significant advantages. In addition, they are simpler than deformation analyses. However, because the actual stress-strain relations are not used in the analysis, limit equilibrium methods do not result in calculation of ex-

pected deformations. Deformations are controlled by designing for an appropriate factor of safety. At least to this extent, the use of limit equilibrium methods of analysis is semiempirical. The factor of safety cannot be measured except when the slope is failing, in which case it is known to be unity. As noted previously, there are many instances in which the precise deformations in a slope are of little concern, provided the material stays in place. In such cases, it is entirely appropriate to undertake analyses with limit equilibrium methods; however, regardless of the complexity of the method employed, the empirical nature of the design criteria should be borne in mind.

LIMIT EQUILIBRIUM ANALYSIS

Limit equilibrium methods for slope stability analysis have been used for several decades, and numerous techniques have been developed. Limit equilibrium analysis is used in design to determine the magnitude of the factor of safety. When a slope has failed, however, the factor of safety is unity, and the analysis can then be used to estimate the average shearing resistance along the failure surface or along part of the failure surface if the shearing resistance is assumed to be known along the remainder. Regardless of the specific procedure for carrying out the computations, the following principles are common to all methods of limit equilibrium analysis.

1. A slip mechanism is postulated. This is done without any major kinematic restriction except that the mechanism be feasible and sensible. In the simpler configurations, the slopes are assumed to fail along planes or circular sliding surfaces. When conditions are not uniform, more complex shapes are known to be appropriate, and analyses have been developed to handle surfaces of arbitrary shape.

2. The shearing resistance required to equilibrate the assumed slip mechanism is calculated by means of statics. The physical concepts used here are that the potential slip mass is in a state of limiting equilibrium and that the failure criterion of the soil or rock is satisfied everywhere along the proposed surfaces. Various methods differ in the degree to which the conditions for equilibrium are satisfied, and some common methods of analysis violate conditions of static equilibrium. This is an important factor in evaluating the rigor of any method.

3. The calculated shearing resistance required for equilibrium is compared with the available shear strength. This comparison is made in terms of the factor of safety, which will be defined more precisely below.

4. The mechanism with the lowest factor of safety is found by iteration. For example, if it is assumed that the slip surface is circular, a search is made for the critical slip circle. When the position of the slip surface is dictated by a dominant weakness, such as sheared clay at residual strength, other trials are unnecessary.

The factor of safety should be defined as clearly as possible and its role understood. The definition used here is as follows:

The factor of safety is that factor by which the shear strength parameters may be reduced in order to bring the slope into a state of limiting equilibrium along a given slip surface.

According to this definition, the factor of safety relates to the strength parameters and not to the strength itself, which in the case of an effective stress analysis depends on the effective normal stress as well. Moreover, this definition implies that the factor of safety is uniform along the entire slip surface.

Other definitions of the factor of safety are also used in geotechnical engineering. For example, the factor of safety is commonly defined as the ratio of a disturbing moment to a restoring moment when the moment equilibrium about the center of rotation of a slip circle is under consideration. This definition is not convenient when slip surfaces are noncircular and therefore is of limited applicability. It is equivalent to the definition given above in the case of a $\phi = 0$ analysis. When the bearing capacity of a foundation is calculated, the factor of safety is usually determined as the ratio of the maximum bearing capacity to the allowable bearing capacity. Used in this way, the factor of safety is a ratio of loads and differs from the factor of safety applied to strength parameters. This is one reason why the magnitude of the factor of safety commonly adopted in design for bearing capacity differs substantially from the values used in slope stability analyses. Occasionally, the factor of safety is also taken as the ratio of the calculated stress at a point to the allowable stress. This definition can only be applied when the stress distribution in the slope is evaluated in detail, and it bears no simple relation to the definition adopted in limit equilibrium analysis. As noted by Barron (7.2), even the definition given here is restrictive because it is based on an assumption for the likely stress path to failure. However, the implications of whether failure might occur under drained or undrained conditions and whether the soil may have a metastable structure can be taken into account by adjusting the factor of safety accordingly.

Partial safety factors are sometimes used in stability analysis. A simple case is the application of separate factors of safety to two strength parameters, for example, c'/F_1 and $\tan \phi'/F_2$, where F_1 and F_2 have different values. Uncertainty about other terms in the stability equation might also be included by application of partial safety factors or coefficients to loads applied to the slope, water pressures, and other parameters. This practice has been common in some parts of Europe (7.29) and recently has been used in probabilistic methods of design.

An understanding of the role of the factor of safety is vital in the rational design of slopes. One well-recognized role is to account for uncertainty and to act as a factor of ignorance with regard to the reliability of the items that enter into analysis. These include strength parameters, pore-pressure distribution, and stratigraphy. In general, the lower the quality of the site investigation is, the higher the factor of safety should be, particularly if the designer has only limited experience with the materials in question. Another use of the factor of safety is to provide a measure of the average shear stress mobilized in the slope. This should not be confused with the actual stresses. A realistic stress distribution cannot, of course, be calculated from a limit equilibrium analysis because stress-strain relations are not used in the analysis. As indicated earlier, a major role of the factor of safety is that it constitutes the empirical tool whereby deformations are limited to tolerable amounts within economic restraints. In this way, the choice of the factor of safety is

greatly influenced by the accumulated experience with a particular soil or rock mass. Since the degree of risk that can be taken is also much influenced by experience, the actual magnitude of the factor of safety used in design will vary with material type and performance requirements. These and related topics influencing the design of stable slopes are discussed in Chapter 8.

Total and Effective Stress Analyses

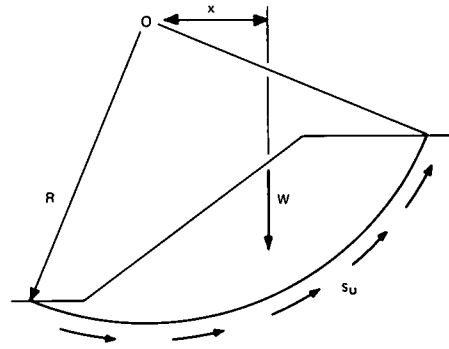
When a fully saturated soil is sheared to failure without permitting drainage, the soil behaves as though it is purely cohesive (i.e., $\phi = 0$ and $s = s_u$) and the results are interpreted in terms of total stresses. This behavior is more properly viewed as a response to the type of test and method of interpretation than as a reflection of basic strength properties. Nevertheless, simple, practical analyses result when $\phi = 0$ behavior is assumed, and, provided the available shearing resistance is selected with care, reasonable results can be obtained. Undrained total stress analysis is limited to slopes where the pore pressures are governed by total stress changes and little time has elapsed so that no significant dissipation has occurred. This is the so-called end-of-construction class of problems. Even in the case of normally or lightly overconsolidated soils, the factor of safety under undrained conditions may not be critical during the life of a slope, as Bishop and Bjerrum (7.4) point out. The case of the failure along the slope of the Kimola Canal in Finland is particularly instructive in this regard (7.45, 7.48).

When pore pressures are governed by steady seepage configurations, stability analysis should be performed in terms of effective stresses. This is the usual condition for natural slopes in both soil and rock. Moreover, all permanent cuts or fills should be analyzed for long-term conditions to see whether these control the design. Some jointed and fissured clays respond to drainage along dominant discontinuities so quickly that it is doubtful that a $\phi = 0$ idealization is ever representative (7.24). These materials should be analyzed in terms of effective stresses regardless of the time of loading or unloading.

It is also possible to analyze slopes under undrained conditions in terms of effective stresses, but the pore pressures must be obtained either by prediction in terms of total stress changes and pore pressure parameters or from field observations. Embankments on soft clay and silt foundations are commonly designed by the use of a $\phi = 0$ analysis and monitored during construction in terms of effective stress. If design has been based on a factor of safety of 1.5 in terms of a total stress analysis, it does not follow that the factor of safety during construction should also be 1.5 in terms of an effective stress analysis. For a given problem the two coincide only at failure, in which case the factor of safety is unity.

A $\phi = 0$ analysis requires knowledge of only the undrained strength, while an effective stress analysis is based on c' , ϕ' , and the pore-pressure distribution. The selection of strength parameters is a key factor in analysis, and this is made difficult by the fact that the relevant parameters are often anisotropic, depend on rate of test and size, and decay with time. Some of the problems associated with selecting appropriate parameters are discussed by Skempton and Hutchinson (7.66) and in Chapter 6. The analytical capabilities discussed

Figure 7.1. Geometry used in slip circle analysis for $\phi = 0$ case.



in the following sections, particularly when facilitated by computer programs, invariably exceed our ability to identify the appropriate slip mechanism, determine representative shear strength parameters, and specify the correct water pressure distribution.

Total Stress Analysis of Soil Slopes ($\phi = 0$)

Homogeneous Isotropic

Figure 7.1 shows a uniform isotropic purely cohesive slope. From considerations of moment equilibrium it can be shown that

$$F = R \int s_u ds / Wx \quad [7.1]$$

where

F = factor of safety,
 ds = increment of arc length,
 s_u = undrained shear strength,

and the other terms are specified in Figure 7.1. This problem is statically determinate, and it is possible to solve for the critical circle as a function of the slope angle and geometry (7.68). The solution takes the form

$$F = N(s_u / \gamma H) \quad [7.2]$$

where

γ = unit weight of the soil,
 H = height of the slope, and
 N = stability factor.

N is a function only of the slope angle (β) and the depth factor (n_d). Charts presenting the results of analytical studies are given by Taylor (7.68, 7.69) and Terzaghi and Peck (7.71).

Nonhomogeneous Isotropic

Normally consolidated clays characteristically exhibit a linear increase of strength with depth. This often is ex-

pressed as the ratio of the undrained strength (s_u) to the vertical effective stress (p) under which the material has been consolidated. Although the ground close to the surface may have been overconsolidated by desiccation, this will be neglected in the analysis presented here. A linear increase in undrained strength with depth can be treated in analysis by assuming that the soil consists of a sequence of layers chosen to represent the variation of strength with depth to an appropriate degree of accuracy. Since this is cumbersome, Gibson and Morgenstern (7.27) sought an analytical solution to the problem. They showed that the factor of safety for a cut in a normally consolidated soil may be found from the following equation, which is analogous to Equation 7.2:

$$F = N(s_u/p)_n [1 - \gamma_w/\gamma] \quad [7.3]$$

where

γ_w = unit weight of water, and
 N = stability factor that depends only on the slope angle (β).

The variation of N with β is shown in Figure 7.2. The factor of safety is independent of the height of the slope. In theory, failure occurs along an infinite number of slip circles that emerge above the toe, and the need to consider a depth factor is obviated. In practice, the position of the actual slip circle will be dictated by some small variation from the assumed strength-depth relation. Further theoretical support for the use of slip circles for this class of problems has been provided by Booker and Davis (7.6), who show, from a more exact analysis based on the theory of plasticity, that the slip circle analysis provides acceptable results for slopes steeper than 5° , i.e., within the usual working range.

Hunter and Schuster (7.38) considered the stability of a cut in soil with a strength profile shown in Figure 7.3. The undrained shear strength increases with depth, but there is a finite intercept at the surface. Both finite and infinite depths of soil were taken into consideration and showed that

$$F = (s_u/p) (\gamma'/\gamma) N \quad [7.4]$$

where γ' is the submerged unit weight of the soil and N depends on β and M . M is a parameter used to specify the strength of the soil at the surface of the ground and is given by

$$M = (h/H)(\gamma_w/\gamma') \quad [7.5]$$

where H is the height of slope, and the other terms are specified in Figure 7.3. The results for an unlimited depth of clay are shown in Figure 7.4, and charts describing the influence of the depth factor are included in the original reference.

Design charts have also been constructed to facilitate the stability analysis of embankments on soils subjected to undrained loading. Nakase (7.60) summarizes the development of various solutions and presents in a concise form the results of the analysis for the case shown in Figure 7.5. This is particularly useful in that it embraces a variety of possible embankment loadings on a soil whose strength increases with depth from a finite intercept at the surface. Both finite and infinite depths of subsurface clay are included in the

analysis. Although the strength of the embankment material is neglected, this assumption is often realistic for reasons of strain compatibility between the fill and the foundation materials. In any case, the assumption leads to conservative results. The final design charts are too numerous to be included here.

A comprehensive set of design charts for the stability of an embankment on a uniform homogeneous purely cohesive foundation was prepared by Pilot and Moreau (7.62). In this case the strength of the embankment material is taken into account, but the foundation is characterized by a uniform undrained strength. The depth to a rigid stratum is varied as well. An extensive analysis over the range of all the parameters of interest has been undertaken, and the factor of safety is presented directly in terms of these parameters.

Anisotropic

The undrained strength of a soil is commonly anisotropic.

Figure 7.2. Variation of stability number with slope angle for cuts in normally consolidated clays (7.27).

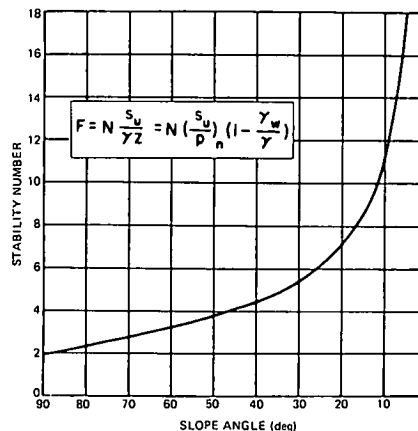
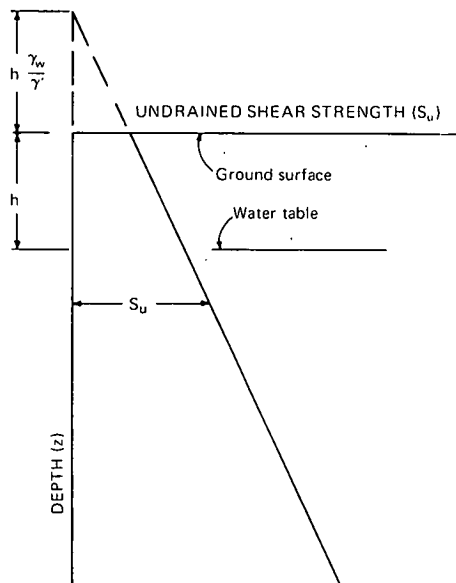


Figure 7.3. Idealized strength-depth relation for normally consolidated clay (7.38).



This can arise from the development of an anisotropic fabric during deposition and subsequent consolidation. Even if anisotropic structure can be ignored, anisotropic states of stress during consolidation result in anisotropy in the undrained strength (7.30). Casagrande and Carrillo (7.8) proposed that the undrained strength varies with direction according to

$$C_i = C_2 + (C_1 - C_2) \cos^2 i \quad [7.6]$$

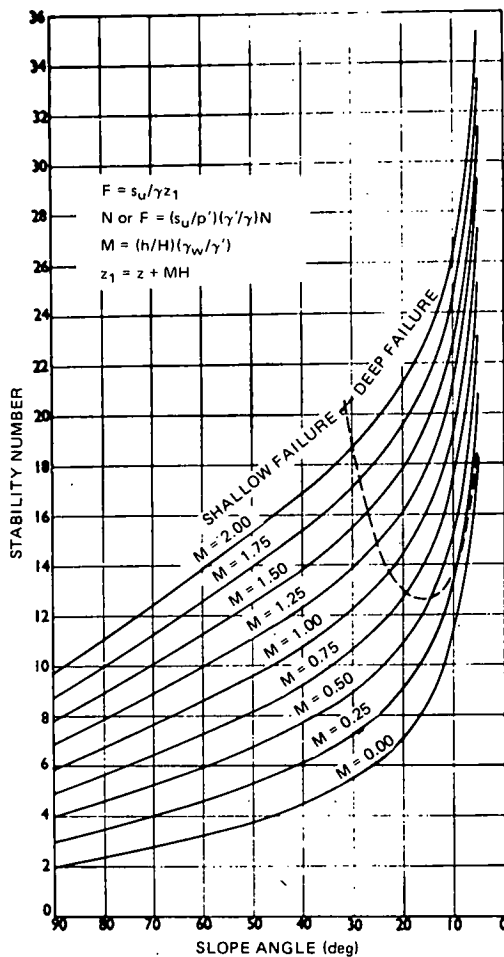
where

- i = angle of deviation from vertical of major principal stress at failure,
- C_i = directional strength in i direction,
- C_1 = strength of vertical specimen, and
- C_2 = strength of horizontal specimen.

There is some experimental support for Equation 7.6 and, since the major principal stress rotates from the vertical toward the horizontal around a slip circle, design charts for the stability of slopes in anisotropic soils are of practical interest. Based on Equation 7.2,

$$F = N(C_1/\gamma H) \quad [7.7]$$

Figure 7.4. General solution for cuts in unlimited depths of normally consolidated clays (7.38).



where N depends on C_2/C_1 and the slope angle (β). Lo (7.53) undertook the evaluation of N for uniform soils and for profiles where the strength increases with depth. For soils conforming to Equation 7.6, the factor of safety obtained by taking anisotropy into account is significantly lower than that from conventional analysis, except for steep slopes. The design charts presented by Lo allow this to be assessed readily.

Method of Slices

When the surface profile or the stratigraphy is irregular, it is not convenient to seek an analytical solution. Instead, Equation 7.1 is evaluated numerically by the method of slices. Figure 7.6 shows the geometry of a circular surface of sliding as used in that method. F is found from

$$F = (\sum s_{u_i} l_i) / (\sum W_i \sin \alpha_i) \quad [7.8]$$

where i denotes each slice in turn, and \sum is the summation of all slices. By use of Equation 7.8, an almost arbitrary degree of complexity can be considered, including anisotropy and variation of strength with depth. It is common to undertake these analyses with a digital computer and many programs exist. For illustrative purposes, the factor of safety is calculated below for a sample problem. The assumptions for the problem are that

1. The soil is saturated,
2. No volume change occurs,

Figure 7.5. Idealized problem of embankment loading on cohesive soils. Stability charts are available for this problem (7.60).

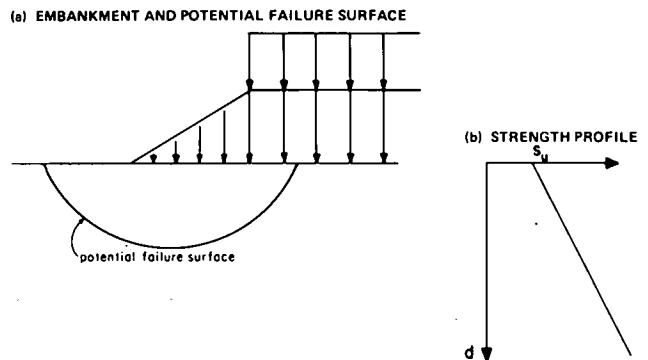
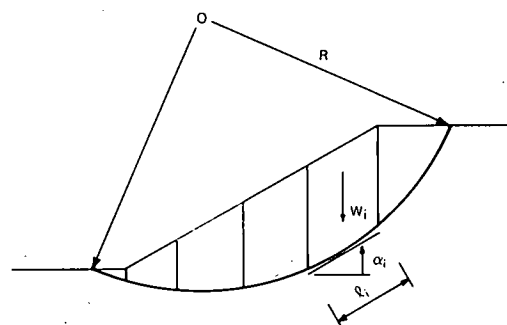


Figure 7.6. Geometry pertaining to a particular circular surface of sliding as used in the method of slices.



3. No dissipation of pore pressure occurs,
4. The mode of failure is slip circle, and
5. The shear strength along the slip surface is the only factor contributing to resistance.

Figure 7.7 shows the cross section of the failure and the force diagram.

The factor of safety is determined by the use of Equation 7.8; i.e., F is the resisting moment divided by the driving moment. The resisting moment is the sum of the mobilized strength (s_u) in each layer times the radius of the circle, and the driving moment is the sum of the weight of each slice times its moment arm from center of rotation. F can be defined this way only because all terms in the resisting moment have s_u .

The calculated dimensions of the slices shown in Figure 7.7 are given in Table 7.1. The remaining calculations are

1. Sum of the driving moments = 29 507 kN·m;
2. Resisting moments = $24 (25 \times AB + 15 \times BC + 35 \times CD) = 24 (25 \times 7.5 + 15 \times 4 + 35 \times 34) = 34 500$ kN·m; and
3. For this example, $F = 34 500/29 507 = 1.17$.

Figure 7.7. Cross section and force diagram of example slip surface failure.

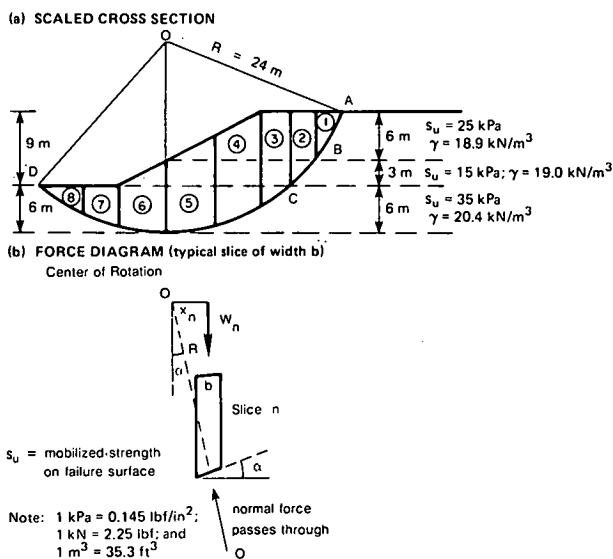


Table 7.1. Calculated dimensions of slices in example.

Slice	Width (m)	Height (m)	Force (kN)	Lever Arm (m)	Driving Moment (kN·m)
1	3	3	170	20	3 400
2	3	7.5	426	17.5	7 455
3	3.5	10.5	703	14	9 842
4	6	12	1400	9	12 600
5	6	10	1182	3	3 546
6	6	7	844	-3	-2 532
7	4	4	326	-8	-2 608
8	6	1.5	183	-12	-2 196
Total					29 507

Note: 1 m = 3.3 ft; 1 kN = 2.25 lbf; and 1 kN·m = 0.74 lbf·ft.

The critical slip circle and the minimum F must be found by iteration. Convergence is fastest if O is moved horizontally.

Effective Stress Analysis of Soil Slopes

Planar Slip Surface

When a soil mass moves predominantly in a translational manner, there is little internal distortion and an infinite slope analysis is often representative. In this analysis, the slip surface is assumed to be a plane parallel to the ground surface and the end effects can be neglected. When sliding takes place in the mantle of weathered material with small ratios of depth to length, this type of analysis is often appropriate. The mechanics of sliding along a planar slip surface will be presented in detail, for they are instructive with regard to all effective stress analyses.

Consider the slope and slip surface shown in Figure 7.8. In an effective stress analysis the pore pressure distribution is assumed to be known, and in this case steady seepage is assumed to be parallel to the surface with the groundwater table as indicated. The forces acting on a slice of width b are shown. Since there is no internal distortion and end effects are neglected,

$$dE = dX = 0 \quad [7.9]$$

The weight of a slice W is given by

$$W = \gamma b d \quad [7.10]$$

The total normal force (N) is related to the effective force (N') and pore pressure (u) by

$$N = N' + u\ell \quad [7.11]$$

and

$$u\ell = (\gamma_w h) b \sec \alpha \quad [7.12]$$

where h is the piezometric head acting on the slip surface. The shear force acting along the base of the slice (S) is

$$S = [(c'b \sec \alpha)/F] + (N - u\ell)(\tan \phi'/F) \quad [7.13]$$

where

c' = cohesion intercept in terms of effective stress, and
 ϕ' = angle of internal friction in terms of effective stress.

By resolving W into components parallel and perpendicular to the slope, we get

$$S = W \sin \alpha \quad [7.14]$$

and

$$N = W \cos \alpha \quad [7.15]$$

The S and N forces can be eliminated by combining Equations 7.13, 7.14, and 7.15, and an explicit relation for F can be found as

$$F = (c'/\gamma d) \sec \alpha \operatorname{cosec} \alpha + (\tan \phi'/\tan \alpha) [1 - (\gamma_w h/\gamma d) \sec^2 \alpha] \quad [7.16]$$

F is a function of two dimensionless ratios, $(c'/\gamma d)$ and $(\gamma_w h/\gamma d)$. The latter, which expresses the relation of the magnitude of the pore pressure at a point and the overburden pressure at that point, has been called the pore-pressure ratio (r_u) by Bishop and Morgenstern (7.5). When r_u is used, Equation 7.16 can be rewritten as

$$F = (c'/\gamma d) \sec \alpha \operatorname{cosec} \alpha + (\tan \phi'/\tan \alpha) (1 - r_u \sec^2 \alpha) \quad [7.17]$$

which in the case of zero cohesion becomes

$$F = (\tan \phi'/\tan \alpha) (1 - r_u \sec^2 \alpha) \quad [7.18]$$

Analyses using Equation 7.17 or 7.18 are readily performed by hand. Several cases exist to illustrate the utility of this analysis in practice (7.39, 7.66).

Circular Slip Surface

Rotational slides occur in many types of soil, and to analyze slopes in these materials by assuming a circular slip surface is common practice. Taylor (7.68) presents stability charts for uniform slopes characterized by both friction and cohesion. However, since these charts are in terms of total stresses, they are of limited value. For heterogeneous conditions, the friction circle method of analysis used by Taylor is less practical than methods that use slices. Therefore, only various methods of slices will be discussed here.

One method of analysis that is accurate for most purposes is that advanced by Bishop (7.3). The forces acting on a typical slice are shown in Figure 7.9. Moment equilibrium about the center of rotation gives

$$\Sigma Wx = \Sigma SR \quad [7.19]$$

Substituting for S in Equation 7.19 and using the Mohr-Coulomb failure criterion in terms of the factor of safety give

$$F = (R/\Sigma Wx) \Sigma [c'\ell + (P - u\ell) \tan \phi'] \quad [7.20]$$

The equilibrium of a slip circle in a frictional medium is statically indeterminate. Attempts to eliminate P from Equation 7.20 by using force equilibrium conditions introduce E or X terms, and an assumption must be made to render the problem statically determinate. In what is usually termed the simplified Bishop method, force equilibrium of a slice in the vertical direction is taken, and the variation in X forces across a slice is ignored. This is tantamount to assuming zero shear between slices and ignoring the requirement for equilibrium in the horizontal direction. The resulting equation for the factor of safety is

$$F = \frac{\Sigma \{ [c'b + (W - ub) \tan \phi'] (1/m_\alpha) \}}{\Sigma W \sin \alpha} \quad [7.21]$$

where

$$m_\alpha = \cos \alpha [1 + (\tan \alpha \tan \phi'/F)] \quad [7.22]$$

Since F appears on both sides of Equation 7.21, it is most convenient to solve for F in an iterative manner. Initially a value of F is assumed and the right side of the equation is evaluated to find a new value. The magnitudes of m_α are changed accordingly, and another iteration is undertaken. The process converges rapidly. Hand calculations are made convenient by the use of tabular forms, such as that shown in Figure 7.10. It is useful to have m_α plotted for any assumed value of F, as shown in Figure 7.11. Several versions of efficient computer programs are available for the solution of Equation 7.21 (7.52, 7.73).

A simpler but less satisfactory method of analysis may be derived by neglecting the internal forces and assuming that

$$N = W \cos \alpha \quad [7.23]$$

Substituting this relation into the equation for overall moment equilibrium results in the following expression for F:

$$F = \Sigma [c'\ell + (W \cos \alpha - u\ell) \tan \phi'] / (\Sigma W \sin \alpha) \quad [7.24]$$

Figure 7.8. Forces acting on idealized slice of infinite slope.

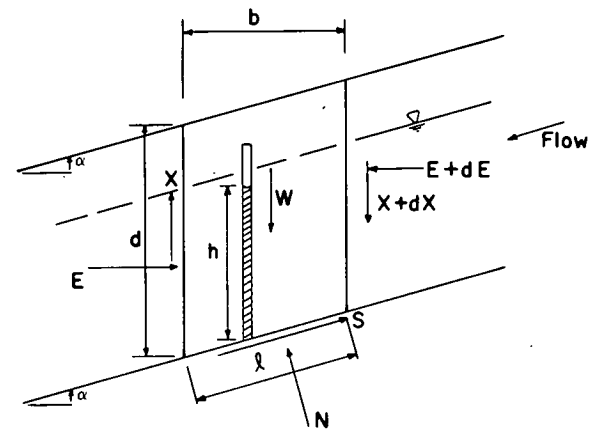


Figure 7.9. Forces involved in effective stress slip circle analysis.

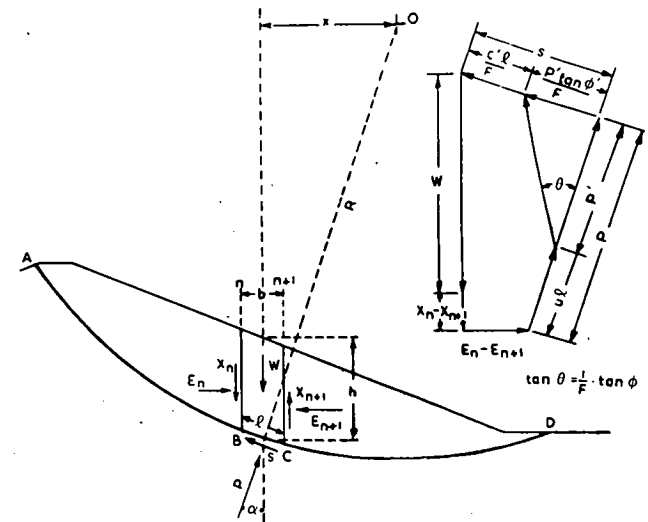


Figure 7.10. Tabular form for computation of factor of safety if surface of sliding is circular and interslice forces are neglected (7.71).

Values from cross section								
	1	2	3	4	5	6	7	8
Slire No.	α°	$\sin \alpha$	W'	$W' \sin \alpha$	$c + \frac{W'}{b} \tan \phi$	$(5) \cdot b$	$F_o = \frac{m_\alpha}{F_o}$	$(6)/(7)$
								$\Sigma(8)$
For first trial, $F_o = \frac{\Sigma(6)}{\Sigma(4)}$ $F = \frac{\Sigma(8)}{\Sigma(4)}$								

Figure 7.11. Values of m_α for use in calculation of factor of safety if surface of sliding is circular and interslice forces are neglected (7.71).

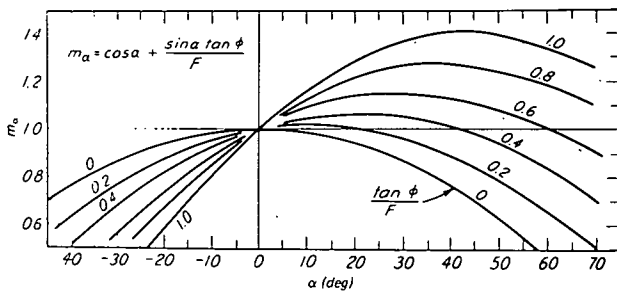
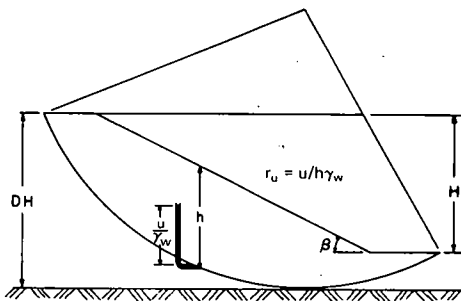


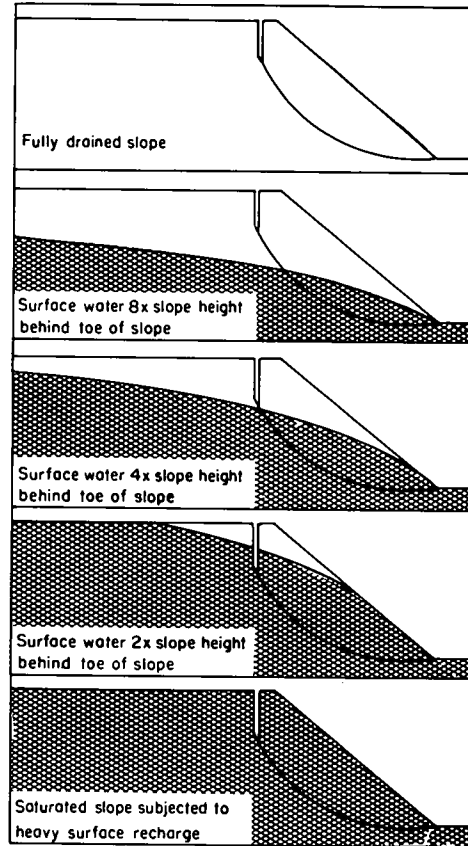
Figure 7.12. Illustration of parameters for use in Bishop-Morgenstern method of analysis (7.5).



This method has been called the Fellenius method, the U.S. Bureau of Reclamation method, the common method of slices; and the ordinary method. As confirmed in practice (7.64), it results in factors of safety that are too low. Although simplicity may have been a reasonable argument at one time for using Equation 7.24, the widespread use of computers precludes algebraic complexity as an excuse for not adopting methods of analysis that are superior from the point of view of mechanics.

Bishop and Morgenstern (7.5) sought to develop charts that would facilitate the use of Equation 7.21 for problems formulated in terms of effective stress. For the uniform slope shown in Figure 7.12, they showed that Equation 7.21 could be cast into the dimensionless form

Figure 7.13. Idealized cases of seepage in slopes for which chart solutions are available (7.37).



$$F = \Sigma \left\{ \left[\frac{c'}{\gamma H} \right] \left(\frac{b}{H} \right) + \left(\frac{b}{H} \right) \left(\frac{h}{H} \right) \times (1 - r_u) \tan \phi' \right\} \left(\frac{1}{m_\alpha} \right) \div \left[\Sigma \left(\frac{b}{H} \right) \left(\frac{h}{H} \right) \sin \alpha \right] \quad [7.25]$$

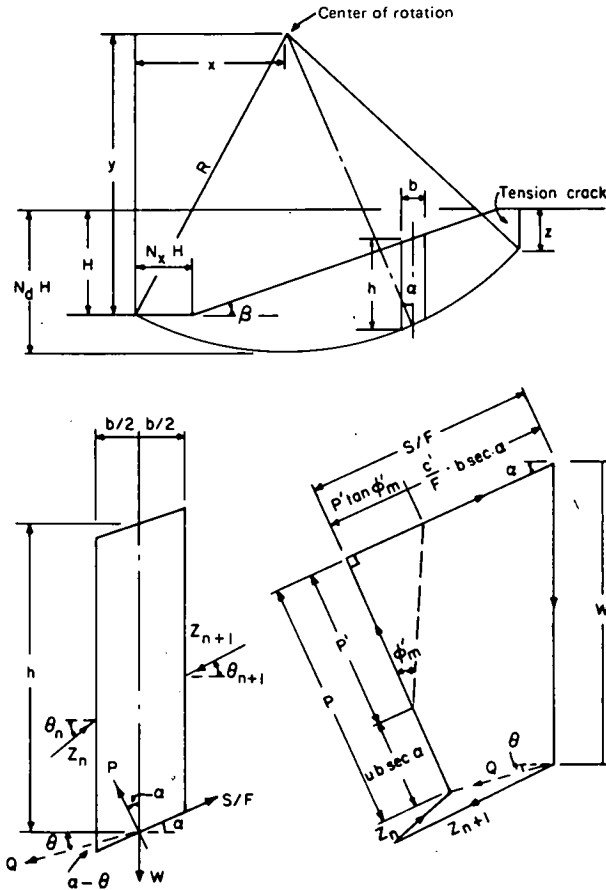
Hence, given $(c'/\gamma H)$, ϕ' , and r_u , F depends only on geometry. They further showed that, for a given geometry expressed by $\cot \beta$ and D and for given soil properties expressed by $(c'/\gamma H)$ and ϕ' , the factor of safety (F) decreased in a linear manner with an increase in the pore-pressure ratio (r_u). This relation is given by

$$F = m - n r_u \quad [7.26]$$

where m and n are called stability coefficients and are presented as functions of the parameters and ratios noted above. The stability coefficients were evaluated over a wide range of values for $(c'/\gamma H)$, ϕ' , β , and D , and the results were presented in both chart and tabular form for ease of use. For a given problem, reference to the appropriate chart or linear interpolation between charts allows one to extract the appropriate values of m and n . The factor of safety is given directly by Equation 7.26. In many cases that arise in practice, r_u is not uniform, and an average value has to be evaluated in order to use the stability coefficients. Guidance in calculating an average value of r_u is given (7.5). When r_u is zero, stability coefficient m gives the same factor of safety as Taylor's charts for a material without cohesion.

Other stability charts that may be found useful have been prepared by Hoek and Bray (7.37) for each of the cases

Figure 7.14. Slip surface dimensions and slice forces for use in Spencer's analysis (7.67).

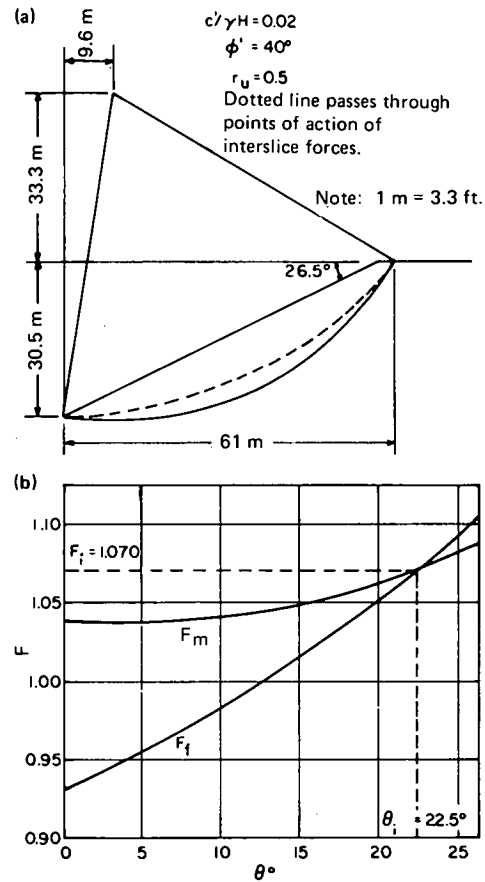


shown in Figure 7.13. These cases represent failure along a circular slip surface in a soil mass subjected to steady seepage and containing a tension crack. The magnitudes of $(c'/\gamma H)$, ϕ' , and β are needed to enter a particular chart representing one of the cases shown in Figure 7.13. The factor of safety is then readily found. The charts are based on a form of friction circle analysis that tends to underestimate the factor of safety. Although they are convenient to use for preliminary design, final design should be checked with a more rigorous analysis.

As pointed out previously, the simplified Bishop method of analysis is adequate for circular slip surfaces, but it assumes that no shear is mobilized between slices and it does not satisfy the condition of force equilibrium in the horizontal direction. Why the method works can be demonstrated by reference to the method presented by Spencer (7.67). Spencer followed the usual procedures in the method of slices (Figure 7.14) and argued that the action of the internal forces acting on a slice can be replaced by their resultant (Q) inclined at θ to the horizontal and passing through the midpoint of a thin slice in order to satisfy moment equilibrium. In general, both Q and θ will vary from slice to slice, and from force equilibrium it can be shown that

$$Q = [(c'b \sec \alpha / F) + (\tan \phi' / F) \times (W \cos \alpha - ub \sec \alpha) - W \sin \alpha] \div \cos(\alpha - \theta) [1 + (\tan \phi' / F) \tan(\alpha - \theta)] \quad [7.27]$$

Figure 7.15. Examples of variation of F_m and F_f with θ in Spencer's analysis (7.67).



For overall equilibrium, the sum of the horizontal components and the vertical components of the interslice forces must each be zero; that is,

$$\Sigma[Q \cos \theta] = 0 \quad [7.28]$$

and

$$\Sigma[Q \sin \theta] = 0 \quad [7.29]$$

Moreover, if the sum of the moments of the external forces about the center of rotation is zero, the sum of the moments of the interslice forces about the center of rotation is also zero. For a circular slip surface,

$$\Sigma[Q \cos(\alpha - \theta)] = 0 \quad [7.30]$$

If θ is assumed constant in order to make the problem determinate, Equations 7.28 and 7.29 reduce to

$$\Sigma Q = 0 \quad [7.31]$$

In either Equation 7.30 or 7.31, the value of F will depend on the assumed constant value of θ . For all but one value of θ , the safety factor determined (Equation 7.31) by force equilibrium (F_f) will not equal the safety factor determined (Equation 7.30) by moment equilibrium (F_m). This is shown in Figure 7.15 by the intersection of curves of F_f and

F_m plotted against values of θ . This intersection gives the pair of values of θ and F that satisfy both equations simultaneously. If θ is set equal to zero and substituted into Equation 7.30, the equation governing Bishop's simplified method (Equation 7.21) is recovered. Therefore, Bishop's simplified method represents the solution for F_m with θ equal to zero. In general, the equation governing moment equilibrium is rather insensitive to variations in θ , and errors of only a few percent result. For the assumption that θ equals zero used in the Bishop simplified method, these errors are not conservative with respect to actual values of θ greater than zero.

Noncircular Slip Surface

When the distribution of shearing resistance within an earth mass becomes nonuniform, slip can occur along surfaces more complex than a circle. Since the shape of the failure surface will be controlled to a large degree by the departure from uniformity, one may wish to analyze the stability along surfaces of arbitrary shapes. Noncircular analysis has proved useful in a number of cases of actual slides (7.22, 7.40).

Morgenstern and Price (7.57) developed a method of analysis that treats a slip surface of arbitrary shape and satisfies all equilibrium requirements. To make the analysis statically determinate, a relation between the internal forces is assumed of the form

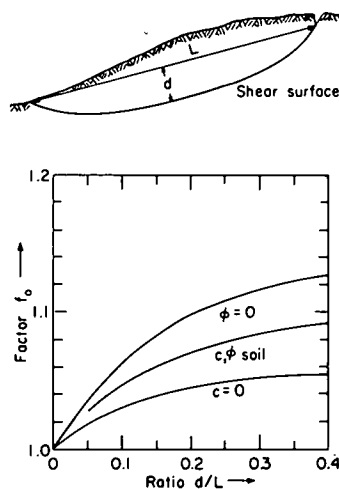
$$X = \lambda f(x) E \quad [7.32]$$

where

λ = factor to be determined in the solution, and
 $f(x)$ = arbitrary function concerning the distribution of internal forces.

The choice of $f(x)$ is limited by conditions of physical admissibility, which require that no tension be developed in the earth mass and that the failure criterion not be violated. Subject to these conditions, the factor of safety is rather insensitive to variations in $f(x)$. A digital computer is needed to solve resulting Equations 7.30 and 7.31. The Spencer analysis of a slip circle discussed previously is equivalent to setting $f(x)$ equal to unity.

Figure 7.16. Curves for use in Janbu's simplified analysis (7.42, 7.43).



A convenient approximate method of analysis suitable for hand calculations and sufficiently accurate for many practical purposes is described by Janbu, Bjerrum, and Kjaernsli (7.43) and Janbu (7.42). This method follows the method of slices and is based on a more elaborate procedure described in detail in the latter reference. The factor of safety is given by

$$F = f_0 \frac{\sum \{ [c'b + (W - ub) \tan \phi'] / \cos \alpha m_\alpha \}}{\sum W \tan \alpha} \quad [7.33]$$

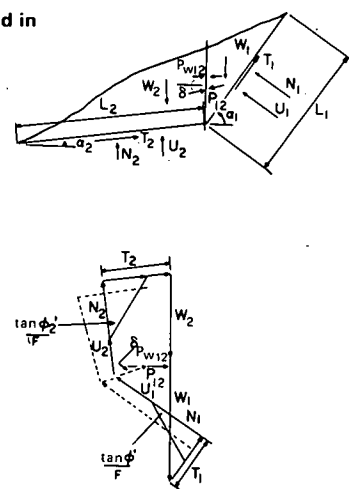
where m_α is defined in Equation 7.22 and used as shown in Figure 7.10. The term f_0 , which is a correction factor for the role of the internal forces, is a function of the curvature of the slip surface and the type of soil; recommended values for this factor are given in Figure 7.16.

Another approximation procedure for analyzing slip along a noncircular sliding surface is the wedge analysis. In this method the potential sliding mass is separated into a series of wedges, and the equilibrium of each wedge is considered in turn. Only the conditions of horizontal and vertical equilibrium are used in the analysis, which is often performed in a graphical manner. Moreover, an assumption must be made regarding the inclination of the force transmitted across the interface between any two wedges. The forces utilized in this method of analysis are shown in Figure 7.17 for a rock mass separated into two wedges; the notation is defined below.

- W_1, W_2 = weight of wedge,
- U_1, U_2 = resultant water pressure acting on base of wedge,
- N_1, N_2 = effective force normal to base,
- T_1, T_2 = shear force acting along base of wedge,
- L_1, L_2 = length of base,
- α_1, α_2 = inclination of base to horizontal,
- P_{w12} = resultant water pressure at interface,
- P_{12} = effective force at interface, and
- δ = inclination of P_{12} to horizontal.

Although the rock mass shown in Figure 7.17 is separated into two wedges, the analysis can be extended readily to any number of wedges. The factor of safety is varied so that the force polygons constructed for each of the wedges satisfy horizontal and vertical equilibrium. A value of δ is

Figure 7.17. Forces used in typical wedge analysis.



assumed, and an initial value of F is taken; this allows the force polygon for the first wedge to be constructed, as shown by the dashed line in Figure 7.17. With the value of P_{12} obtained in this manner, the force polygon for the second wedge is readily constructed. This will not close, in general, and the value of F must be varied until it does. The final solution also is given in Figure 7.17.

The factor of safety is sensitive to the assumed value of δ . Conservative values of the factor of safety may be found by setting δ equal to zero. The maximum possible value of δ is that value which is compatible with failure in the direction of the interface, but this usually leads to an overestimate of the factor of safety. Experience indicates that putting δ between 10° and 15° usually gives reasonable results.

Pore-Pressure Distribution

To perform a stability analysis in terms of effective stress requires that the pore pressures be specified. A distinction is made between problems of undrained and drained loading. In the first case, the magnitude of the pore pressure is influenced to a large degree by the changes in total stress to which the earth mass is subjected; in the second case, the pore-pressure distribution is controlled by steady seepage conditions. When an analysis of a soil loaded under undrained conditions is made in terms of effective stress, the pore pressures must be either predicted or measured. The confidence level for predicting pore pressures in cuts is not high; however, it is somewhat better in the case of foundations beneath embankments because the total stress change is less influenced by the initial state of stress in the ground. For a clay soil, the initial stresses in the ground are not usually known to a high degree of accuracy. In general, the pore pressure (u) that is used in a stability analysis is given by

$$u = u_0 + \Delta u \quad [7.34]$$

where

u_0 = pore pressure before any stress change, and
 Δu = change in pore pressure due to change in total stress.

The change in pore pressure (Δu) can be estimated approximately if the pore-pressure coefficients and the changes in total stresses are known. In turn, the change in total stress at a point may be estimated from stress analyses. Linear elasticity is commonly assumed; therefore, the results are of limited applicability, particularly if the factor of safety approaches unity and a substantial volume is permitted to yield. In the case of cuts, the total stresses decrease so that with time the pore pressures increase from their end-of-construction values, thereby resulting in reduction of the factor of safety.

Although it is not common to predict pore-pressure changes in slopes due to changes in total stress, it is common to measure them on major projects and to use the data directly in stability analyses. Instrumentation for measuring pore pressures is discussed in Chapter 5. In the case of embankments on soft impervious foundations, monitoring pore pressures is almost routine. However, this monitoring may not be adequate to avert a slip (7.1).

Steady seepage through a soil is governed by the Laplace equation, for which a variety of methods are available to find solutions subject to appropriate boundary conditions (7.9, 7.31). These techniques are equally useful in the design of drainage measures if the influence of the design on the magnitude and distribution of the pore pressures must be ascertained. In some cases, analytical methods will have the capability to provide solutions in closed form, but more generally the problems that arise in practice are analyzed by constructing flow networks graphically by the use of electric analogue techniques or numerically by means of the finite element method (7.33, 7.46, 7.70).

An understanding of hydrogeologic controls is important in the solution, either by analysis or by observation, of any steady seepage problem. In particular, it is essential to recognize the control that regional groundwater systems have on the local pore-pressure distribution in a natural slope or excavation. The recognition of groundwater discharge areas and an identification of the recharge areas are a starting point for almost all seepage studies, either computational or observational. High pore pressures are often found in discharge areas associated with topographic lows, and they may have a controlling influence on the stability of slopes. Analyses of regional groundwater systems have been made by Toth (7.72) and Freeze and Witherspoon (7.25, 7.26), and their significance for slope stability problems is noted by Patton and Deere (7.61).

Heterogeneity of soil deposits has a marked influence on the pore-pressure distribution under steady seepage conditions. Kenney and Chan (7.47) describe a detailed field study of the anisotropy of the permeability in a varved soil. The ratio of the permeability of flow parallel to the varves to that perpendicular to the varves was found to be less than five. Although this is less than might be expected, it still exerts a significant influence on the pressure distribution that develops during steady seepage conditions. Eigenbrod and Morgenstern (7.22) report a striking example of the influence of geologic detail on the pore-pressure distribution within a slope about 20 m (70 ft) high in Cretaceous bedrock overlain by till. This slope was shown to possess at least three water tables, the upper two of which were perched on bentonitic layers. Moreover, no pore pressures acted along much of the failure surface, since it lay on a fractured free-draining coal layer. Careful field observations guided by an appreciation for the role of geologic details are needed in cases like this if misleading results are to be avoided.

Analysis of Rock Slopes

The analysis of the stability of a rock slope is normally undertaken in terms of effective stress because the permeability of rock masses is usually so high that undrained loading does not arise. To perform an analysis requires that the shear strength parameters, the pore-water pressures, and the sliding mechanism be specified. In hard rocks, the sliding mechanism is controlled by structural features, and a detailed study of those features is a prerequisite for rational design of a rock slope. Further guidance on the evaluation of rock structure characteristics is given in Chapter 9.

The analysis of a rock slope in terms of a factor of safety is a subordinate activity to achieving a clear understanding

of the controlling geology and water-pressure configuration. In the case of slopes in soil, the confidence limits attached to the application of the total process of investigation, testing, and analysis are determined by the successful explanation of well-documented case histories (7.66). However, in the case of rock slides, few such case histories report completely the geologic configuration, give good quality test data, and have been subjected to an acceptable analysis. Moreover, the shearing resistance in rocks is often sensitive to movements. Small movements can result in substantial decreases in available shearing resistance and hence in the factor of safety. Therefore, the confidence limits of design for a given factor of safety are generally lower for rock slopes than for soil slopes. In other words, rock slopes designed for a given factor of safety have a higher degree of risk associated with them than most soil slopes designed for the same factor of safety. If the degree of risk is to be the same, it is prudent to design rock slopes for higher factors of safety, unless only the residual shearing resistance is being mobilized. However, design for a higher factor of safety is not always economically feasible.

The shearing resistance of rock is more variable than that of soil, and the same is true of the water-pressure distribution. Both the strength and the water-pressure distribution are dominated to a large degree by the pattern of discontinuities in the rock mass. In the case of strength, the usual procedures for investigation involve in situ testing or sampling and testing, as discussed in Chapter 4. The water-pressure distribution can only be evaluated by field investigations coupled with an appreciation of the hydrogeologic constraints.

Morgenstern (7.55) reviews the influence of groundwater on the stability of rock slopes and shows how flow through a discrete network of discontinuities can be replaced by flow through an imaginary medium of equivalent permeability. If the crack spacing is small compared to the size of the discontinuity, conventional procedures for solving problems of flow through porous media are applicable. If this is not the case, the attitudes and hydraulic conductivities of the actual discontinuities should be considered (7.54, 7.74). This is difficult to do, and considerable judgment is required to properly evaluate the water pressure distribution. Patton and Deere (7.61) stress the large local differences in water pressures that can arise in a jointed rock mass and the large fluctuations that can develop as a consequence of rainfall. They also note the significant effects that faults can have on groundwater conditions. Those effects can be either beneficial or deleterious, depending on the attitude and characteristics of the fault gouge. Groundwater flow around faults and other similar features requires special study in slope-stability analyses. The evaluation of water pressures in a rock mass is made more difficult by the sensitivity of the hydraulic conductivity to small deformations. A small amount of slip along a discontinuity can result in a disproportionate change in hydraulic conductivity (7.65). Although methods of analysis can be developed to take these effects into consideration (7.7, 7.56), the information needed to conduct these analyses is only rarely available.

Stability analyses are usually concerned with sliding. However, discrete blocks of rock can also rotate outward or topple. A tendency to topple depends on the shape of the block and is usually readily discernible in the field. The

condition for toppling is dictated by the position of the weight vector of the block relative to the base of the block. If the weight vector passing through the center of gravity of the block falls outside the base of the block, toppling will occur; this is shown in Figure 7.18 (7.37).

A common case of instability in rock slopes is sliding along a planar surface. The analysis of this case is a simple extension of the stability of a rough rigid block on an inclined plane, and, for the case shown in Figure 7.19a, we have

$$F = \frac{(2c/\gamma H) P + [Q \cot \psi_p - R(P + S)] \tan \phi}{(Q + RS \cot \psi_p)} \quad [7.35]$$

where

$$P = [1 - (Z/H)] \operatorname{cosec} \psi_p, \quad [7.36]$$

$$Q = \{ [1 - (Z/H)]^2 \cot \psi_p - \cot \psi_f \} \sin \psi_p, \quad [7.37]$$

$$R = (\gamma_w/\gamma) (Z_w/Z) (Z/H), \text{ and} \quad [7.38]$$

$$S = (Z_w/Z) (Z/H) \sin \psi_p. \quad [7.39]$$

But if the tension crack is in the surface above the slope (Figure 7.19h),

$$Q = \{ [1 - (Z/H)]^2 \cos \psi_p (\cot \psi_p \tan \psi_f - 1) \} \quad [7.40]$$

should be used. All other terms are as shown in the figure. Hoek and Bray (7.37) undertook a systematic study of the influence of the various parameters that enter into Equation 7.35 and prepared design charts to facilitate its use. If a planar mechanism is appropriate, it is also possible to assess the influence of drainage and other stabilizing measures, such as berms or cables, by using simple extensions of Equation 7.35.

Often two or three sets of discontinuities intersect, and sliding of a wedge of rock becomes possible. In general, differing amounts of friction and cohesion can act on the planes, and the water pressure acting normal to the surfaces bounded by the planes should be taken into consideration. A comprehensive analysis of the mechanics of wedge failure is given by Hendron, Cording, and Aiyer (7.32). Figure 7.20 shows a tetrahedron bounded by two base planes, which may be intersecting joint sets. Failure may occur by sliding along the line of intersection of the two planes or by sliding on either one of the two planes. General procedures have been developed for determining the factor of safety graphically by vector analysis by the use of stereo networks and numerically by hand or by computer (7.32, 7.37). The following steps are included in the analysis.

1. The intersections of the various joint sets with one another and with the slope face are inspected to determine the tetrahedra that may be potential failure wedges; these are then analyzed in detail.

2. The forces tending to disturb the equilibrium of the wedge are added vectorially to give the resultant driving force. These disturbing forces arise from the weight of the

Figure 7.18. Conditions for sliding and toppling of rock block on inclined surface (7.37).

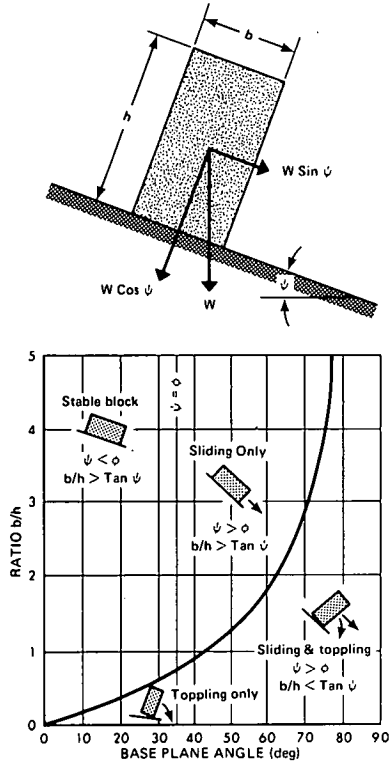


Figure 7.19. Rock-slope failure due to sliding along single discontinuity (7.37).

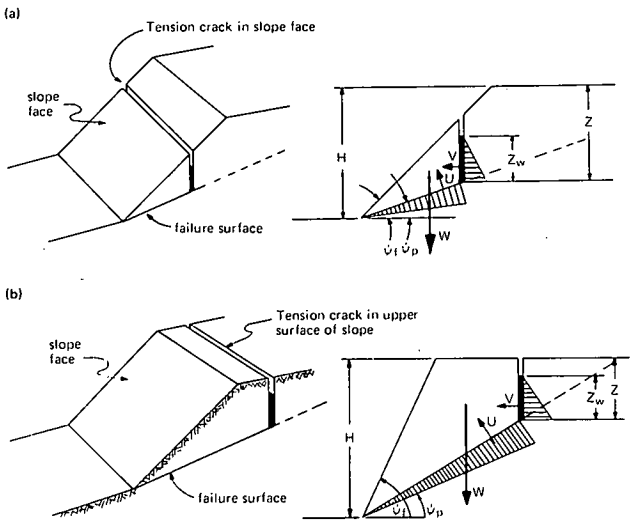
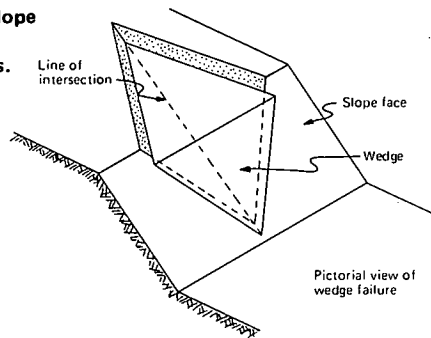


Figure 7.20. Rock-slope failure by sliding on intersecting joint sets.



wedge, any external load applied to the wedge, and the pore-water forces acting on the various faces.

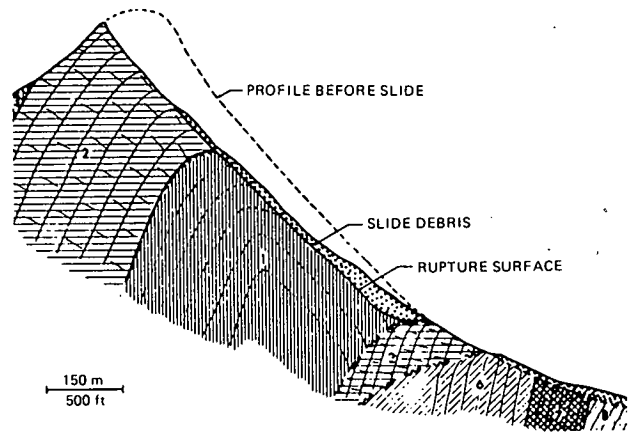
3. The mode of failure is then determined. For example, a wedge supported on two base planes can either slide along the line of intersection of the two planes, slide on either plane, or rotate on either plane. The kinematics of failure will depend on the orientation of the disturbing force in relation to the orientation of the supporting planes. Various kinematic tests can be defined (7.32).

4. After the mode of failure is determined, the maximum shearing resistance that can be mobilized in the direction of movement is compared to the shearing forces necessary for equilibrium in order to compute a factor of safety.

Hendron, Cording, and Aiyer (7.32) point out that, in the common case of a wedge resting on two base planes and acted on only by its own weight, sliding will occur along the line of intersection of the two planes if a line drawn down the dip in both planes tends to intersect the line of intersection. If in either one of the planes a line drawn down the dip is directed away from the line of intersection, sliding will occur on that plane only and the wedge will move away from the line of intersection. If a wedge is acted on only by its own weight, it will slide down the maximum dip if sliding occurs on one plane, and the factor of safety can be readily computed. If sliding occurs along the line of intersection in the absence of pore pressures, the angle of shearing resistance required for equilibrium will always be equal to or less than the slope of the line of intersection. The actual factor of safety will depend on the relative attitudes of bounding planes. Stability charts for some frictional wedges without water-pressure loading and with sliding along the line of intersection have been prepared by Hoek and Bray (7.37). Stereographic projection is a most useful tool for the analysis of the stability of rock wedges. Both friction and cohesion as well as water pressures can be taken into account. Designers concerned with rock slopes are well advised to become familiar with these techniques, which are described in the references cited earlier, as well as by John (7.44), Heuze and Goodman (7.34), and Hoek (7.36).

Some rock slides can be treated as two-dimensional problems on noncircular surfaces. Figure 7.21 shows the failure surface of the famous Frank slide (7.13), an example of this type of slide; the Vaiont slide is another example (7.59).

Figure 7.21. Cross section of Frank slide (7.13).



These cases are amenable to analysis by the use of the same methods adopted for noncircular slides in soil.

DEFORMATION ANALYSIS

The use of deformation analysis in the design of slopes is an attractive concept; however, practical applications of deformation analysis are limited at present. Until the last decade, this limitation was imposed by the absence of suitable analytical methods to perform deformation analysis. Few elastic solutions were available for boundary value problems similar to slopes, and the solutions available were not well suited to practical design. The development of the finite element method has largely eliminated this impediment. As engineers have taken advantage of this analytical tool, however, other equally vexing problems have been recognized, and these still severely limit the practical application of the finite element method in most slope design problems.

In principle, a deformation analysis, particularly an analysis utilizing the finite element method, must include the following characteristics.

1. The stress field must satisfy equilibrium at every point. The prediction of this equilibrium stress field is usually done by use of elastic theory to describe the stresses and deformations, but plasticity or other material models might also be used. To predict a stress field requires that the stress-strain relation for the soil be known.

2. Boundary conditions of stress and deformation must be satisfied. These characteristics can be contrasted to the principles of limit equilibrium noted in the earlier section on principles of limit equilibrium analysis.

The lack of success in applying deformation analysis in the design of slopes can be attributed directly to the difficulties associated with the stress-strain relation noted above. Soil is generally nonlinear, nonuniform, inelastic, and anisotropic. Each of these characteristics must be idealized and used in a deformation analysis, and the difficulty in describing natural soil deposits in these terms is the major factor that limits application of the finite element method in the analysis of slopes. The problem is aggravated by our lim-

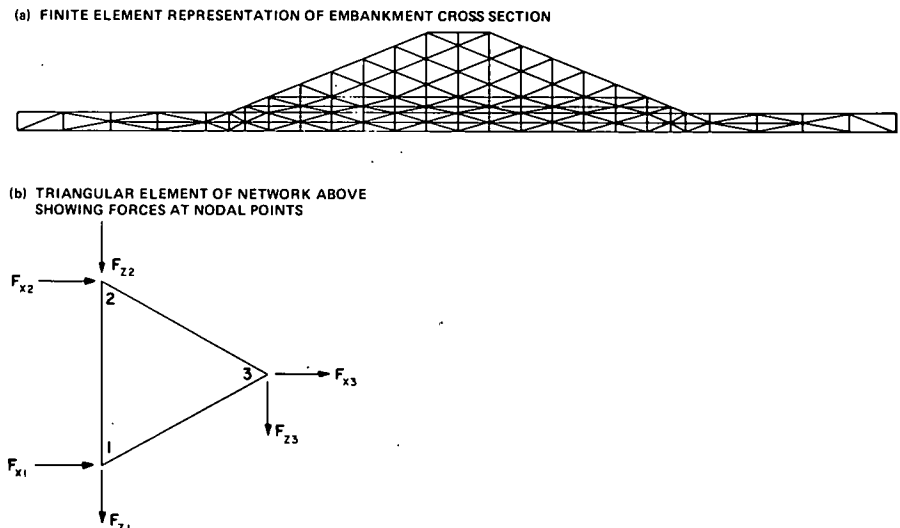
ited knowledge of the in situ stresses before excavation. Under special conditions, particularly in analysis of dams and embankments of compacted earth fill, the method is being used successfully (7.51); but the deformation analysis of natural soils is primarily a research activity at present and has little practical application. A review of the principles of the finite element method is nevertheless appropriate because some practical applications are being made at present and progress in developing practical applications in the future is anticipated.

The finite element method has been applied to geotechnical engineering problems since the late 1960s, having been developed a decade earlier for applications in structural engineering and continuum mechanics. Since that time, extensive literature has developed, including several texts. An important and useful reference and summary of this work was compiled by Desai (7.16); this volume is recommended as a starting point for those interested in the method.

The underlying principle of the finite element method is that the behavior of a complex continuum can be approximated by the collective behavior of parts representing the continuum. The parts are selected so that their individual behaviors can be described simply. By requiring adjacent elements to behave similarly at selected points of contact or nodes, the overall continuum is modeled.

In a typical application (Figure 7.22a), an embankment deformation analysis begins by using a group of simple elements to approximate the fill. In a two-dimensional analysis, either triangular or quadrilateral elements are commonly used. For a two-dimensional analysis, equations describing the behavior of these elements in plane strain are used in contrast to the plane stress formulation often used for problems in structural engineering (7.18). A three-dimensional problem can be modeled by using solid elements. For each element in the model, a series of equations and an appropriate stress-strain relation, such as elasticity, are used to relate the application of loads to deformation at the nodes. In the direct method of element formulation (Figure 7.22b), a triangular element with nodes only at the corners can be used. Other elements in the problem are formulated in a similar fashion. The deformation of adjacent elements is then constrained to be identical at the nodal points. There

Figure 7.22. Finite element idealization of embankment.



is no requirement that deformations or stress be compatible between other than nodal points on adjacent elements in this example. As a result, this solution would be only an approximation of the actual system behavior. A more accurate approximation might result by adding nodes at the midpoint of each side of the element. Variational methods of element formulation also add to the power, accuracy, and flexibility of the procedure (7.17).

After a model of the embankment is formulated, appropriate loading and deformation boundary conditions are imposed. In geotechnical engineering problems, one of the important loads is the self-weight of the material, which can be represented by applying a vertical force at each node (7.21). Several studies have shown, however, that this method has limitations in predicting deformations. An alternate approach is to perform a sequential analysis in which the problem is solved in successive stages to represent a series of steps in construction or excavation (7.20).

The procedures described above lead to the assembly of a large number of simple equations, and the solution of these equations requires use of a digital computer. Finite element programs applicable to geotechnical engineering problems are readily available from several sources and the literature. Because continuing improvements are made in these programs, no specific references are given here.

The use of finite element methods to predict the stability of slopes does not offer any improvement over accurate limit equilibrium methods (7.19). Its usefulness lies in deformation analysis, in which accuracy is directly related to the stress-strain behavior of the soil. Active research continues to be directed toward this subject. Modeling the complex behavior of soil has progressed from initial efforts using linear elasticity to include many of the more complicated elements of the constitutive relation. Linear elasticity is almost always a poor model of actual soil behavior, but it does have the advantages of low cost and simple application (7.19). In some cases, simple linear models can be used to determine qualitatively the form of particular slope or embankment deformations. However, this method can lead to significant errors and should be used with caution. Another application of linear elasticity is described by Cole and Burland (7.12), who used finite element analysis to extrapolate from measured field performance. They used field measurements to calibrate their analysis and by trial and error selected the best values of the linear stress-strain relation to match their measurements. These constitutive relations were then used to predict subsequent loads and deformations during further construction involving the same material.

Several other forms of the basic stress-strain relation have been proposed (7.19). Most methods deal with nonlinear elastic forms, such as bilinear (7.20) or hyperbolic (7.49). Multilinear and spline functions (7.15, 7.23) have been used to define the constitutive relation in even greater detail. The theory of plasticity (7.63) offers great potential through the use of yield functions to describe nonlinear effects, together with the coupling of volume changes to stress changes. In the use of elastic theory, volume changes are modeled by a variable Poisson's ratio. Because of the relative incompressibility of undrained saturated soils, Poisson's ratio has a constant value of approximately 0.5. This number may be changed slightly (e.g., to 0.49) to eliminate computational

instability in some programs. When volume changes do occur (for example, during drainage or in partially saturated soils), the variation in Poisson's ratio can be related experimentally to stress changes (7.50) or to the coefficient of earth pressure at rest (K_0). Consolidation effects during loading can also be considered (7.10, 7.41). Some advantages have been shown (7.11) for using the shear modulus (G) and bulk modulus (K) instead of Young's modulus (E) and Poisson's ratio (ν) for formulating elastic finite element problems in soil.

During deformation analysis, it is common to exceed the available strength of soil at specific points long before overall collapse of the slope is imminent. This presents the problem of modeling local yield. Since many soils exhibit strain-softening behavior after yield (Chapter 6), it is appropriate to model this by use of finite element methods. Höeg (7.35) describes one method using an elastic-plastic model, and several other techniques have been proposed (7.17).

Modeling natural deposits of rock and heavily overconsolidated clay requires consideration of joints and similar discontinuities. These features can experience shear translation along the joints, as well as dilatancy and opening of the joints during movement of the slope. The application of the finite element method to rock (7.75) and particularly to dilating jointed rock (7.28) has relevance to slope-stability problems.

Other characteristics of the constitutive behavior of soil and rock have been considered for application to finite element deformation analysis. These include anisotropy (7.14), creep, and tensile cracking (7.23). As greater and greater complexity is added to the modeling parameters, the cost of the finite element analysis increases, often beyond practical limits. However, the major problem limiting the application of deformation analysis to natural slopes and cuts remains the heterogeneous and complex nature of the materials involved.

REFERENCES

- 7.1 Al-dhahir, Z., Kennard, M. F., and Morgenstern, N. R. Observations on Pore Pressures Beneath the Ask Lagoon Embankments at Fiddler's Ferry Power Station. In *In Situ Investigations in Soils and Rocks*, British Geotechnical Society, London, 1970, pp. 265-276.
- 7.2 Barron, R. A. Correspondence. *Geotechnique*, Vol. 14, No. 4, 1964, pp. 360-361.
- 7.3 Bishop, A. W. The Use of the Slip Circle in the Stability Analysis of Slopes. *Geotechnique*, Vol. 5, No. 1, 1955, pp. 7-17.
- 7.4 Bishop, A. W., and Bjerrum, L. The Relevance of the Triaxial Test to the Solution of Stability Problems. Proc., Research Conference on Shear Strength of Cohesive Soils, Boulder, American Society of Civil Engineers, New York, 1960, pp. 437-501.
- 7.5 Bishop, A. W., and Morgenstern, N. R. Stability Coefficients for Earth Slopes. *Geotechnique*, Vol. 10, No. 4, 1960, pp. 129-150.
- 7.6 Booker, J. R., and Davis, E. H. A Note on a Plasticity Solution to the Stability of Slopes in Homogenous Clays. *Geotechnique*, Vol. 22, No. 4, 1972, pp. 509-513.
- 7.7 Brekke, T. L., Noorishad, J., Witherspoon, P. A., and Maini, Y. N. T. Coupled Stress and Flow Analysis of Fractured Dam Foundations and Rock Slopes. Proc., Symposium on Percolation Through Fissured Rock, International Society of Rock Mechanics and International Association of Engineering Geology, Stuttgart, Deutsche Gesellschaft für Erd- und Grundbau, Essen, West Germany, 1972, pp. T4-J, 1-8.

- 7.8 Casagrande, A., and Carrillo, N. Shear Failure of Anisotropic Materials. In *Contributions to Soil Mechanics: 1941-1953*, Boston Society of Civil Engineers, 1944, pp. 122-135.
- 7.9 Cedergren, H. R. *Seepage, Drainage, and Flow Nets*. Wiley, New York, 1967, 489 pp.
- 7.10 Christian, J., and Boehmer, J. W. Plane Strain Consolidation by Finite Elements. *Journal of Soil Mechanics and Foundations Division, American Society of Civil Engineers*, New York, Vol. 96, No. SM4, 1970, pp. 1435-1457.
- 7.11 Clough, R. W., and Woodward, R. J. Analysis of Embankment Stresses and Deformations. *Journal of Soil Mechanics and Foundations Division, American Society of Civil Engineers*, New York, Vol. 93, No. SM4, 1967, pp. 529-549.
- 7.12 Cole, K. W., and Burland, J. B. Observations of Retaining Wall Movements Associated With A Large Excavation. Proc., 5th European Conference on Soil Mechanics and Foundation Engineering, Spanish Society for Soil Mechanics and Foundations, Madrid, 1972, pp. 327-342.
- 7.13 Cruden, D. M., and Krahn, J. A Reexamination of the Geology of the Frank Slide. *Canadian Geotechnical Journal*, Vol. 10, No. 4, 1973, pp. 581-591.
- 7.14 D'Appolonia, D. J., and Lambe, T. W. Method for Predicting Initial Settlement. *Journal of Soil Mechanics and Foundations Division, American Society of Civil Engineers*, New York, Vol. 96, No. SM2, 1970, pp. 523-544.
- 7.15 Desai, C. S. Nonlinear Analyses Using Splice Functions. *Journal of Soil Mechanics and Foundations Division, American Society of Civil Engineers*, New York, Vol. 97, No. SM10, 1971, pp. 1461-1480.
- 7.16 Desai, C. S., ed. *Applications of the Finite Element Method in Geotechnical Engineering: A Symposium*. U.S. Army Engineer Waterways Experiment Station, Vicksburg, Miss., 1972, 1227 pp.
- 7.17 Desai, C. S. Theory and Applications of the Finite Element Method in Geotechnical Engineering. In *Application of the Finite Element Method in Geotechnical Engineering: A Symposium* (Desai, C. S., ed.), U.S. Army Engineer Waterways Experiment Station, Vicksburg, Miss., 1972, pp. 3-90.
- 7.18 Desai, C. S., and Abel, J. F. Introduction to the Finite Element Method: A Numerical Method for Engineering Analysis. Van Nostrand-Reinhold, New York, 1972, 477 pp.
- 7.19 Duncan, J. M. Finite Element Analysis of Stresses and Movements in Dams, Excavations, and Slopes. In *Application of the Finite Element Method in Geotechnical Engineering: A Symposium* (Desai, C. S., ed.), U.S. Army Engineer Waterways Experiment Station, Vicksburg, Miss., 1972, pp. 267-326.
- 7.20 Dunlop, P., and Duncan, J. M. Development of Failure Around Excavated Slopes. *Journal of Soil Mechanics and Foundations Division, American Society of Civil Engineers*, New York, Vol. 96, No. SM2, 1970, pp. 471-493.
- 7.21 Dunlop, P., Duncan, J. M., and Seed, H. B. Finite Element Analyses of Slopes in Soil. Department of Civil Engineering, Univ. of California, Berkeley, Rept. TE-68-3, 1968.
- 7.22 Eigenbrod, K. D., and Morgenstern, N. R. A Slide in Cretaceous Bedrock at Devon, Alberta. In *Geotechnical Practice for Stability in Open Pit Mining* (Brawner, C. O., and Milligan, V., eds.), Society of Mining Engineers, American Institute of Mining, Metallurgical and Petroleum Engineers, New York, 1972, pp. 223-238.
- 7.23 Eisenstein, Z., Krishnappa, A. V., and Morgenstern, N. R. An Analysis of the Cracking at Duncan Dam. Proc., Specialty Conference on Performance of Earth and Earth-Supported Structures, Purdue Univ., Lafayette, Ind., American Society of Civil Engineers, New York, Vol. 1, Part 1, 1972, pp. 765-778.
- 7.24 Esu, F. Short-Term Stability of Slopes in Unweathered Jointed Clays. *Geotechnique*, Vol. 16, No. 4, 1966, pp. 321-328.
- 7.25 Freeze, R. A., and Witherspoon, P. A. Theoretical Analysis of Regional Groundwater Flow: 1-Analytical and Numerical Solutions to the Mathematical Model. *Water Resources Research*, Vol. 2, No. 4, 1966, pp. 641-656.
- 7.26 Freeze, R. A., and Witherspoon, P. A. Theoretical Analysis of Regional Groundwater Flow: 3-Quantitative Interpretations. *Water Resources Research*, Vol. 4, No. 3, 1968, pp. 581-590.
- 7.27 Gibson, R. E., and Morgenstern, N. R. A Note on the Stability of Cuttings in Normally Consolidated Clays. *Geotechnique*, Vol. 12, No. 3, 1962, pp. 212-216.
- 7.28 Goodman, R. E., and Dubois, J. Duplication of Dilatancy in Analysis of Jointed Rocks. *Journal of Soil Mechanics and Foundations Division, American Society of Civil Engineers*, New York, Vol. 98, No. SM4, 1972, pp. 399-422.
- 7.29 Hansen, J. B. The Philosophy of Foundation Design: Design Criteria, Safety Factors, and Settlement Limits. In Proc. (Vesic, A. S., ed.), Symposium on Bearing Capacity and Settlement of Foundations, Duke Univ., Durham, N.C., 1965, 1967, pp. 9-14.
- 7.30 Hansen, J. B., and Gibson, R. E. Undrained Shear Strengths of Anisotropically Consolidated Clays. *Geotechnique*, Vol. 1, No. 3, 1949, pp. 189-204.
- 7.31 Harr, M. E. *Groundwater and Seepage*. McGraw-Hill, New York, 1962, 315 pp.
- 7.32 Hendron, A. J., Jr., Cording, E. J., and Aiyer, A. K. Analytical and Graphical Methods for the Analysis of Slopes in Rock Masses. U.S. Army Engineer Nuclear Cratering Group, Livermore, Calif., Technical Rept. 36, 1971.
- 7.33 Herbert, R., and Rushton, K. R. Ground-Water Flow Studies by Resistance Networks. *Geotechnique*, Vol. 16, No. 1, 1966, pp. 53-75.
- 7.34 Heuze, F. E., and Goodman, R. E. Three-Dimensional Approach for Design of Cuts in Jointed Rock. In *Stability of Rock Slopes* (Cording, E. J., ed.), Proc., 13th Symposium on Rock Mechanics, Univ. of Illinois at Urbana-Champaign, American Society of Civil Engineers, New York, 1972, pp. 397-404.
- 7.35 Höeg, K. Finite Element Analysis of Strain-Softening Clay. *Journal of Soil Mechanics and Foundations Division, American Society of Civil Engineers*, New York, Vol. 98, No. SM1, 1972, pp. 43-58.
- 7.36 Hoek, E. Methods for the Rapid Assessment of the Stability of Three-Dimensional Rock Slopes. *Quarterly Journal of Engineering Geology*, Vol. 6, No. 3, 1973, pp. 243-255.
- 7.37 Hoek, E., and Bray, J. W. *Rock Slope Engineering*. Institution of Mining and Metallurgy, London, 1974, 309 pp.
- 7.38 Hunter, J. H., and Schuster, R. L. Stability of Simple Cuttings in Normally Consolidated Clays. *Geotechnique*, Vol. 18, No. 3, 1968, pp. 372-378.
- 7.39 Hutchinson, J. N. A Landslide on a Thin Layer of Quick Clay at Furre, Central Norway. *Geotechnique*, Vol. 11, No. 2, 1961, pp. 69-94.
- 7.40 Hutchinson, J. N. A Reconsideration of the Coastal Landslides at Folkestone Warren, Kent. *Geotechnique*, Vol. 9, No. 1, 1969, pp. 6-38.
- 7.41 Hwang, C. T., Morgenstern, N. R., and Murray, D. W. Application of the Finite Element Method to Consolidation Problems. In *Application of the Finite Element Method in Geotechnical Engineering: A Symposium* (Desai, C. S., ed.), U.S. Army Engineer Waterways Experiment Station, Vicksburg, Miss., 1972, pp. 739-765.
- 7.42 Janbu, N. Slope Stability Computations. In *Embankment-Dam Engineering* (Hirschfeld, R. C., and Poulos, S. J., eds.), Wiley, New York, 1973, pp. 47-86.
- 7.43 Janbu, N., Bjerrum, L., and Kjaernsli, B. Soil Mechanics Applied to Some Engineering Problems. *Norwegian Geotechnical Institute, Oslo, Publ. 16*, 1956, pp. 5-26.
- 7.44 John, K. W. Engineering Analysis of Three-Dimensional Stability Problems Utilizing the Reference Hemisphere. Proc., 2nd Congress, International Society of Rock Mechanics, Belgrade, Vol. 3, 1970, pp. 385-391.
- 7.45 Kankare, E. Failures at Kimola Floating Canal in Southern Finland. Proc., 7th International Conference on Soil Mechanics and Foundation Engineering, Mexico City, Vol. 2, 1969, pp. 609-616.
- 7.46 Kealy, C. D., and Busch, R. A. Determining Seepage Characteristics of Mill-Tailings Dams by the Finite Element Method. U.S. Bureau of Mines, Rept. RI 7477, 1971, 113 pp.

- 7.47 Kenney, T. C., and Chan, H. T. Field Investigation of Permeability Ratio of New Liskeard Varved Soil. *Canadian Geotechnical Journal*, Vol. 10, No. 3, 1973, pp. 473-488.
- 7.48 Kenney, T. C., and Uddin, S. Critical Period for Stability of an Excavated Slope in Clay Soil. *Canadian Geotechnical Journal*, Vol. 11, No. 4, 1974, pp. 620-623.
- 7.49 Kondner, R. L., and Zelasko, J. S. A Hyperbolic Stress-Strain Formulation for Sands. Proc., 2nd Pan-American Conference on Soil Mechanics and Foundations Engineering, Brazil, American Society of Civil Engineers, New York, Vol. 1, 1963, pp. 289-324.
- 7.50 Kulhawy, F. H., and Duncan, J. M. Stresses and Movements in Oroville Dam. *Journal of Soil Mechanics and Foundations Division, American Society of Civil Engineers*, New York, Vol. 98, No. SM7, 1972, pp. 653-655.
- 7.51 Lefebvre, G., Duncan, J. M., and Wilson, E. L. Three-Dimensional Finite Element Analysis of Dams. *Journal of Soil Mechanics and Foundations Division, American Society of Civil Engineers*, New York, Vol. 99, No. SM7, 1973, pp. 495-507.
- 7.52 Little, A. L., and Price, V. E. The Use of an Electronic Computer for Slope Stability Analysis. *Geotechnique*, Vol. 8, No. 3, 1958, pp. 113-120.
- 7.53 Lo, K. Y. Stability of Slopes in Anisotropic Soils. *Journal of Soil Mechanics and Foundations Division, American Society of Civil Engineers*, New York, Vol. 91, No. SM4, 1965, pp. 85-106.
- 7.54 Louis, C. Strömungsvorgänge in klüftigen Medien und ihre Wirkung auf die Standsicherheit von Bauwerken und Boscungen im Fels. *Institut Bodenmechanik und Felsmechanik, Universitat Fridericiana Karlsruhe*, Vol. 30, 1967, 121 pp.
- 7.55 Morgenstern, N. R. The Influence of Groundwater on Stability. In *Stability in Open Pit Mining* (Brawner, C. O., and Milligan, V., eds.), Society of Mining Engineers, American Institute of Mining, Metallurgical and Petroleum Engineers, New York, 1971, pp. 65-81.
- 7.56 Morgenstern, N. R., and Guthrie, H. Seepage Into an Excavation in a Medium Possessing Stress-Dependent Permeability. Proc., Symposium on Percolation Through Fissured Rock, International Society of Rock Mechanics and International Association of Engineering Geology, Stuttgart, Deutsche Gesellschaft für Erd- und Grundbau, Essen, West Germany, 1972, pp. T2-C, 1-15.
- 7.57 Morgenstern, N. R., and Price, V. E. The Analysis of the Stability of General Slip Surfaces. *Geotechnique*, Vol. 15, No. 1, 1965, pp. 79-93.
- 7.58 Morgenstern, N. R., and Price, V. E. A Numerical Method for Solving the Equations of Stability of General Slip Surfaces. *Computer Journal*, Vol. 9, No. 4, 1967, pp. 388-393.
- 7.59 Müller, L. The Rock Slide in the Vaiont Valley. *Felsmechanik und Ingenieur Geologie*, Vol. 2, No. 3-4, 1964, pp. 148-212.
- 7.60 Nakase, A. Stability of Low Embankment on Cohesive Soil Stratum. *Soils and Foundations*, Vol. 10, No. 4, 1970, pp. 39-64.
- 7.61 Patton, F. D., and Deere, D. U. Geologic Factors Controlling Slope Stability in Open Pit Mines. In *Stability in Open Pit Mining* (Brawner, C. O., and Milligan, V., eds.), Society of Mining Engineers, American Institute of Mining, Metallurgical and Petroleum Engineers, New York, 1971, pp. 23-48.
- 7.62 Pilot, G., and Moreau, M. *La Stabilité des Remblais Sur Sols Mous*. Eyrolles, Paris, 1973.
- 7.63 Prevost, J.-H., and Höeg, K. Effective Stress-Strain-Strength Model for Soils. *Journal of Geotechnical Engineering Division, American Society of Civil Engineers*, New York, Vol. 101, No. GT3, 1975, pp. 259-278.
- 7.64 Sevaldson, R. A. The Slide in Lodalen, October 6, 1954. *Geotechnique*, Vol. 6, No. 4, 1956, pp. 167-182.
- 7.65 Sharp, J. C., and Maini, Y. N. T. Fundamental Considerations on the Hydraulic Characteristics of Joints in Rock. Proc., Symposium on Percolation Through Fissured Rock, Stuttgart, International Society of Rock Mechanics and International Association of Engineering Geology, Deutsche Gesellschaft für Erd- und Grundbau, Essen, West Germany, 1972, pp. T1-F, 1-15.
- 7.66 Skempton, A. W., and Hutchinson, J. N. Stability of Natural Slopes and Embankment Foundations. Proc., 7th International Conference on Soil Mechanics and Foundation Engineering, Mexico, State-of-the-Art Volume, 1969, pp. 291-340.
- 7.67 Spencer, E. A Method of Analysis of the Stability of Embankments Assuming Parallel Inter-Slice Forces. *Geotechnique*, Vol. 17, No. 1, 1967, pp. 11-26.
- 7.68 Taylor, D. W., *Stability of Earth Slopes*. *Journal of Boston Society of Civil Engineers*, Vol. 24, No. 3, 1937, pp. 197-246.
- 7.69 Taylor, D. W. *Fundamentals of Soil Mechanics*. Wiley, New York, 1948, 700 pp.
- 7.70 Taylor, R. L., and Brown, C. B. Darcy Flow Solutions With a Free Surface. *Journal of Hydraulics Division, American Society of Civil Engineers*, New York, Vol. 93, No. HY2, 1967, pp. 25-33.
- 7.71 Terzaghi, K., and Peck, R. B. *Soil Mechanics in Engineering Practice*. Wiley, New York, 1967, 729 pp.
- 7.72 Toth, J. A Theoretical Analysis of Groundwater Flow in Small Drainage Basins. Proc., 3rd Hydrology Symposium, National Research Council of Canada, Ottawa, 1962, pp. 75-96.
- 7.73 Whitman, R. V., and Bailey, W. A. Use of Computers for Slope Stability Analysis. *Journal of Soil Mechanics and Foundations Division, American Society of Civil Engineers*, New York, Vol. 93, No. SM4, 1967, pp. 475-498.
- 7.74 Wittke, W., Rissler, P., and Semprich, S. Three-Dimensional Laminar and Turbulent Flow Through Fissured Rock According to Discontinuous and Continuous Models. Proc., Symposium on Percolation Through Fissured Rock, International Society of Rock Mechanics and International Association of Engineering Geology, Stuttgart, Deutsche Gesellschaft für Erd- und Grundbau, Essen, West Germany, 1972, pp. T1-H, 1-18 (in German).
- 7.75 Zienkiewicz, O. C., Best, B., Dullage, C., and Stagg, K. G. Analysis of Nonlinear Problems in Rock Mechanics With Particular Reference to Jointed Rock Systems. Proc., 2nd Congress, International Society of Rock Mechanics, Belgrade, Vol. 3, 1970, pp. 501-509.