Distribution of Bus Transit On-Time Performance

RICHARD P. GUENTHNER AND KASIMIN HAMAT

One method of improving the effectiveness of a bus system is to improve schedule reliability. In evaluating the effect of different strategies to improve reliability, the distribution of bus arrival times in relation to the scheduled arrival times of the buses (adjusted arrival time) is needed. Consequently, an analysis was made to find the distribution of on-time performance. Data were collected from the Milwaukee County Transit System (MCTS) for this study. The results showed that adjusted bus arrival times tend to follow a gamma distribution. This finding differs from the ones that had been proposed in the past. In addition to this finding, adjusted arrival times were examined in relation to the distance along a route, the location of the peak load point, and the headway. Buses in the morning and evening peaks tend to arrive late. However, the midday buses tend to arrive early.

A business must have a market in order to exist. To have a market, it must provide services or products that meet customers' needs. An insufficient understanding of customers is one of many reasons that a business or industry cannot survive or operate competitively. Regardless of the industry, service, or product, quality control is needed ultimately to satisfy the customers. In any service industry, customers are concerned with convenience, service cost, and service performance. However, one of the most important measures of quality of service performance is on-time performance. That is why on-time performance has been widely used as a slogan or trademark by service industries to promote their services. In addition to this marketing aspect, on-time performance is used to reduce cost. But is on-time performance important in the bus transit industry? From a preliminary survey, Bates (*1*) showed a universal agreement among transit operators that on-time performance is an important aspect of transit operations (see Table 1). His survey also demonstrated strong support for research in this area (see Table 2).

Many factors, such as the availability of seats, crime, and maintenance of vehicles, influence people's decision on whether to use bus transit regularly. However, one very important factor is passenger waiting time. A shorter waiting time will make people more likely to ride buses or to become regular riders.

One way to minimize passenger waiting time is to have reliable bus schedule time adherence. Turnquist (*2*) found that once regular passengers are confident that the bus will arrive on time, they can plan their arrival at the bus stop so as to be there just before the bus arrives.

R.P. Guenthner, Professional Engineering Consultants, P.A., 1440 East English Street, Wichita, Kans. 67211. K. Hamat, Greenfield, New South Wales 2176, Australia.

In addition to attracting more riders, on-time performance is important for planning bus headways. This could result in reductions not only of passenger loading variations but also of operating costs. The relationship between bus headways and passenger loading variations was shown by Shanteau (*3*). He concluded that, if the coefficient of variation of bus headways exceeds about 0.30, unequal headways contribute almost exclusively to the variability of loads on buses. That is, headways are poorly controlled. If this is the case, he suggested that bus operators should invest in control strategies to reduce the variance in headways.

DEFINITION OF ON-TIME PERFORMANCE

On-time performance is defined as a bus arriving, passing, or leaving a predetermined bus stop along its route within a time period that is no more than x minutes earlier and no more than y minutes later than a published schedule time. The values of x and y vary across the transit industry. However, one minute and five minutes are the most common values used informally for x and y, respectively (*1*).

This study's main focus is on-time performance, not reliability. On-time performance differs from reliability. For example, if a period of one minute early to three minutes late is defined as on time, a four-minute-late bus is considered to be not on time. If the bus is always four minutes late, however, the consumer might consider this to be very reliable service.

CAUSES OF POOR ON-TIME PERFORMANCE AND STRATEGIES TO IMPROVE IT

To identify poor on-time performance on any route, data from that route should be collected. The data taker can be stationed at a stop or on the bus. Or automatic vehicle monitoring might be used for less costly, more accurate, and more comprehensive data. The data can then be examined briefly to see whether that route has problems concerning on-time performance.

After the routes or systems with poor on-time performance have been identified, the causes for this need to be explored. First, the nature of the problem needs to be categorized. For example, a route that is consistently late must be treated differently from one that is unpredictably late, early, or on time.

Possible causes for poor on-time performance may be as follows:

Variable Ridership

If the route's ridership, for some reason, has large day-to-day changes, the bus might be found to be early on the days of low ridership and late on peak ridership days.

Increased Ridership

If the ridership has increased since the latest schedule revision, then the bus may be found to be consistently late.

External Factors

One example of this may be at a railroad grade crossing. A long or slow freight train is unpredictable and can cause extremely long delays.

Variable Heavy Traffic

Scheduling can be adjusted around consistent heavy traffic. If traffic conditions are variable, however, then the exact arrival times of buses will be very difficult to guarantee. Bus priority techniques may be a solution.

Lack of Schedule Control

Operators need to be sure that buses stay on time. If they do not, better control is needed.

TABLE 1 HOW IMPORTANT IS ON-TIME PERFORMANCE (*1*)?

	Number	Total %	Adj %
Not Important	1	0.7	0.7
Important	37	25.3	25.9
Moderate Important	1	0.7	0.7
Very Important	50	34.2	34.9
Highly Important	8	5.5	5.7
Extremely Important	12	8.2	8.4
Critical	7	4.8	4.9
Essential	27	18.5	18.9
Subtotal	143	97.9	100.0
No Response	3	2.1	-
Total	146	100.0	-

TABLE 2 IS RESEARCH NEEDED (*1*)?

	Number	Total %	Adj %
Yes	90	61.6	70.3
No	38	26.0	29.7
No Opinion	18	12.4	-
Total	146	100.0	100.0

Impossible Schedule

The times presented on a printed schedule may be unreasonable goals. If the system is otherwise operating on time, revising the schedule may solve the problem.

Depending on the nature of the problem, a number of strategies can be used to improve on-time performance. These may be divided into two general categories: (1) adjusting the schedule to reflect the service and (2) adjusting the service to meet the schedule.

Adjusting the schedule may include changed time points, added schedule slack, or longer or shorter layover periods. Adjusting the service could include increased monitoring, longer or shorter routes, fewer stops, express service, or bus priority treatments, such as exclusive lanes or signal preemption.

Abkowitz and Tozzi (*4*) summarized recent research toward methods to measure, evaluate, and improve service reliability. The reader should refer to this work for further details.

DISTRIBUTION OF ADJUSTED ARRIVAL TIMES

Adjusted arrival time is defined in this paper as the difference between the actual or observed arrival time and the scheduled arrival time at a bus stop along a route. In the present research, the distribution of adjusted arrival times is studied. This distribution can be used to (1) measure on-time performance using a significantly smaller sample size, (2) estimate the probability of a bus being on time, and (3) model passenger waiting times, passenger arrivals, and on-time performance. Different strategies can then be either implemented or simulated to evaluate the effects or potential effects relating to on-time performance.

Turnquist (*2*) developed a methodology for estimating passenger waiting time as a function of the variation of bus arrival times. In developing this theory, he suggested that the distribution of bus arrival times at a point is log normal. Guenthner and Sinha (*5*) later used Turnquist's theories to determine average passenger waiting times as an intermediate step in evaluating performance.

Bates (*1*) found that the determination of on-time performance appears to be a largely informal practice with little statistical basis. Consequently, Talley and Becker (*6*) proposed that the exponential probability distribution be used to compute the probabilities that buses on a particular route arriving at a given bus stop will be more than x minutes early and more than y minutes late. They divided time interval data for buses into two groups: those regarding lateness and those regarding earliness. The Kolmogorov-Smirnov goodness-of-fit was performed to make inference to the null hypothesis that the samples used were taken from an exponential probability distribution. The null hypothesis could not be rejected for the late or early time intervals. Consequently, using the exponential distribution, the probability that buses will be more than b minutes late is expressed as follows:

$$\text{Prob}\ (y > b) = e^{-ab} \quad (1)$$

where

e = the base of natural logarithms,
a = $1/u$, and
u = arithmetic mean of the values of y in the sample.

The same formula is also proposed with respect to early arrivals.

Turnquist (*2*) dealt more with the passenger waiting times, for random and nonrandom arrivals, and with the proportion of nonrandom arrivals than with bus transit on-time performance. However, his work showed the important relationship between service reliability and passenger dwell time. Talley and Becker (*6*) were the first ones who analyzed on-time performance with a statistical basis. Their analysis separated earliness and lateness. On-time buses were included in both samples. This approach is useful in many ways. It has difficulty, however, in terms of predicting the probability of on-time bus arrivals.

In the present study, the samples were not divided into early and late. In addition, data were recorded in seconds, which was deemed to be most accurate since the probability distribution of bus arriving times is a continuous distribution.

ROUTE SELECTION

Data for this study were recorded by observing buses from the Milwaukee County Transit System (MCTS), which serves Milwaukee County, Wisconsin, with a 1980 population of 964,988. This system operates sixty-six routes, nine of which are express. A grid system of routes is used and encouraged by a free one-hour unlimited use transfer. MCTS charges a flat fare.

Routes 10, 23, 30, and 31 were chosen to collect the on-time performance data. These routes were used because all of them pass through downtown Milwaukee and through a common point (12th Street-Wisconsin Avenue). In addition, all the buses of these routes travel a distance long enough to be considered suitable for data collection for the analysis. Bus stops located near the 12th Street-Wisconsin Avenue intersection were the main points selected for collecting the data (see Figure 1). A time point, 12th Street is near the maximum load point for most of the routes. All the buses of the chosen routes, eastbound and westbound, pass by these stops except eastbound buses of Route 10, which pass by the bus stop at 12th Street-Wells Street, one block to the north (see Figure 1).

To examine on-time performance as a function of distance along the route, bus stops at the County Hospital and at Jackson Street-Wisconsin Avenue were also used to collect data for Route 10 analysis only. The stop at Jackson Street-

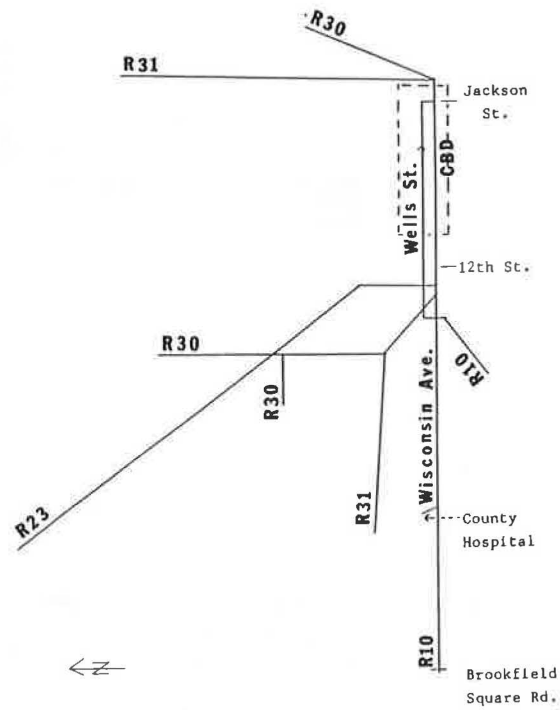

FIGURE 1 Route location map.

Wisconsin Avenue is the east end of Route 10. The County Hospital is the west end of one of two branches of Route 10. Only the arriving times were gathered at these stops. The approximate bus running-time from Jackson Street-Wisconsin Avenue to 12th Street-Wisconsin Avenue is 10 minutes. The time from 12th Street-Wisconsin Avenue to the County Hospital is 23 minutes (see Figure 2).

DATA COLLECTION METHOD

The MCTS defined the time points as the bus leaving times. Toward this end, the procedures described next were used to collect the data.

At intermediate points, the following standards were used: (1) if buses did not stop, the times when the front doors of buses just passed the stop post were recorded; (2) if buses stopped at the bus stop and the traffic signal at the intersection was green, then the times when buses started to move were recorded; and (3) if buses stopped at the bus stop and the intersection traffic signal was red, then the times when buses started to move after the signal turned green were recorded.

At end points of the route, the arriving times were recorded when the bus doors were opened.

Before and after a data collection session, the time of the digital watch used was checked with the central phone time. This phone time is used by Milwaukee County Bus Transit drivers to meet the schedules. The correct published schedules used by the drivers were obtained from the MCTS. The recorded or observed times were subtracted from the scheduled times. The differences were recorded in seconds. A negative time means the bus was early, and a positive time indicates the bus was late.

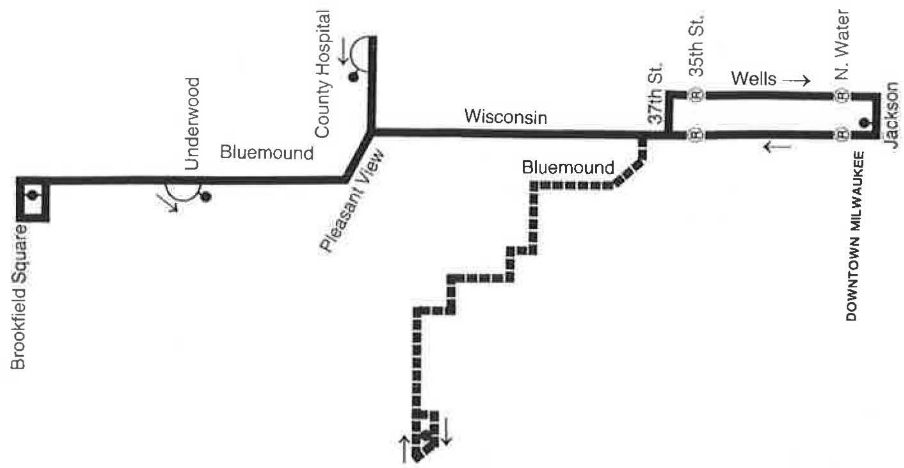

FIGURE 2 Time points and route destinations for Route 10.

Milwaukee's "window" of on-time is from −60 to +180 seconds plus 15 seconds of rounding to −75 to +195 seconds. Consequently, all data within this range should be considered on time.

Data were collected for peak and midday periods. Morning peak extends from 7:00 a.m. to 10:00 a.m., while evening peak is from 3:00 p.m. to 6:00 p.m. Midday data were recorded between 11:00 a.m. and 1:00 p.m. All data were recorded on weekdays.

ANALYSIS OF ON-TIME PERFORMANCE

Frequency and other statistics were used first to analyze the data. Table 3 shows the statistics of each route and of all routes combined. Buses were recorded as early as −522 seconds (−8.7 minutes) and as late as 693 seconds (11.55 minutes).

Figure 3 shows the relationship between the ranges of headways and their related average arriving times. The shortest range of headways (0–5 minutes) has the highest mean (79 seconds) of adjusted arrival times. Then the mean decreases as the headway increases. The mean drops to 18 seconds when the headway ranges from 11 to 15 minutes. It increases sharply, however, when the headway ranges from 16 to 20 minutes. But then, it drops again when the headway is equal 21 minutes or more.

An analysis was also performed to compare the arriving times at different points along Route 10 (see Table 4). The result shows that, on average, eastbound buses arrived much earlier than westbound buses. However, the east end of the route for this part of the analysis is in the downtown area. Table 4 also shows that while buses arrived earlier than the scheduled times at end points, they arrived later than the scheduled times at the 12th Street-Wisconsin Avenue stop, near the peak load.

There are many reasons that buses often arrive earlier than the scheduled times. Traffic may be less congested in areas away from the central business district. And the distance between stops may be longer. These factors enable bus drivers to speed up once they know they are late. In addition, once buses move farther away from a peak load area, fewer pas-

TABLE 3 STATISTICS OF EACH ROUTE AND OF ALL ROUTES COMBINED (ADJUSTED ARRIVAL TIMES)

	Value					Sample
Route	Lowest (second)	Highest (second)	Mean	Median	Std.Dev.	Size
10	-244	545	34	22	130	183
23	-288	598	57	52	151	204
30	-522	617	49	37	134	268
31	-180	693	45	27	149	137
All	-522	693	47	33	140	792

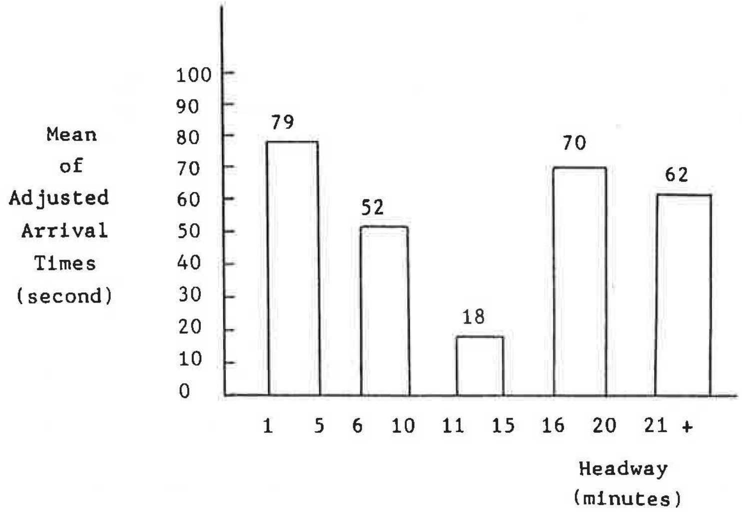

FIGURE 3 Adjusted arrival times as a function of headways.

sengers board the buses. Also, there is a greater pressure on the drivers to be there on time and make the buses available for the next run. One incentive to arrive earlier at the end points is the extra time drivers can have to drink a cup of coffee or to read a newspaper.

The respective performances of the eastbound and westbound buses were compared as shown in Table 5. The means of each peak period and midday period of westbound buses are higher than the ones of eastbound buses. This means that, on average, the westbound passengers waited longer than the eastbound passengers.

The data also show that while buses in morning and evening peaks arrived later than the scheduled times, midday buses arrived earlier than the scheduled times (see Table 6). This could be due to less congested traffic and a smaller volume of passengers in midday.

TABLE 4 COMPARISON FOR DIFFERENT POINTS (ROUTE 10 ONLY)

	County Hospital	12th street	Jackson-Wisconsin
	WB (second)	EB & WB (second)	EB (second)
Mean	-17	34	-63
Std.Dev.	104	130	193

WB - Westbound

EB - Eastbound

TABLE 5 WESTBOUND AND EASTBOUND COMPARISON OF ADJUSTED ARRIVAL TIMES (ALL ROUTES AT 12TH STREET)

	Eastbound (second)			Westbound (second)		
	M.Peak	Midday	E.Peak	M.Peak	Midday	E.Peak
Mean	33	-1	41	71	39	78
Std.Dev.	156	123	155	128	142	118

TABLE 6 COMPARISON OF PEAKS AND MIDDAY OF ADJUSTED ARRIVAL TIMES (ALL ROUTES AT 12TH STREET)

	M.Peak (second)	Midday (second)	E.Peak (second)
Mean	51	-71	60
Std.Dev.	144	134	139

DISTRIBUTION OF ON-TIME PERFORMANCE

Finally, an analysis was made to find the distribution of on-time performance. A USPRP IMSL subroutine was used to find the probability distribution of on-time performance (*7*). This program was used for the purpose of initial screening. The results show that the distribution follows closely the normal distribution from the left tail (early arrivals) only up to a certain point before the right tail. Consequently, this distribution does not appear to fit a normal distribution. Because rarely are buses extremely early but sometimes are extremely late, an appropriate distribution would logically be one with a long right (positive) tail and a short left (negative) tail. Examination of standard continuous distributions indicated that under certain conditions, either a gamma or a log normal distribution fits this description. Since the range of both of these distributions is from zero to infinity, the data were transformed so that the smallest value of the data became zero.

To see whether the data could be represented by either distribution, the Kolmogorov-Smirnov test was performed. One constraint of the Kolmogorov-Smirnov test is that the population parameters cannot be estimated from the sample. Consequently, for this analysis, the odd-numbered observations were used to estimate the parameters, and the even-numbered observations were used for fitting the distribution. This is a standard procedure (*8*).

To perform this test, the largest vertical difference between the theoretical cumulative probability distribution and the actual cumulative probability distribution should be found. This value is then compared to the Kolmogorov-Smirnov Z value, which for an alpha of 0.01 equals $(1.63/n^{0.5})$ if n, the sample size, is greater than 35. These data have a sample size of 396, yielding a Z value of 0.081. For a value greater than the Z value, the null hypothesis that the data are gamma or log normally distributed can be rejected.

The vertical differences were computed between the cumulative probability of the theoretical distributions and the cumulative probability of the actual data (see Table 7). The largest difference for the log normal distribution is 0.0829, indicating that the data are not log normally distributed. However, the largest difference for the gamma distribution is 0.0704. Therefore, the null hypothesis cannot be rejected. The distribution of adjusted bus arrival times appears to be best represented by a gamma distribution.

The theoretical formula of a gamma distribution density function is expressed as follows:

$$P(x) = \frac{\beta^{\alpha}x^{\alpha-1}}{\Gamma(\alpha)}e^{-\beta x} \quad x > 0 \tag{2}$$

TABLE 7 THEORETICAL AND OBSERVED CUMULATIVE PROBABILITY DISTRIBUTIONS

Adjusted Arrival Time (seconds)	Observed	Gamma		Lognormal	
		Theoretical	Difference	Theoretical	Difference
242	0.0050	0.0015	0.0035	0.0003	0.0047
302	0.0100	0.0126	-0.0126	0.0068	0.0032
402	0.0680	0.1116	-0.0436	0.1020	-0.0340
450	0.1920	0.2148	-0.0228	0.2148	-0.0228
500	0.3280	0.3535	-0.0255	0.3632	-0.0352
551	0.4370	0.5074	-0.0704*	0.5199	-0.0829*
601	0.6340	0.6489	-0.0149	0.6628	-0.0288
650	0.7320	0.7642	-0.0322	0.7734	-0.0414
700	0.8230	0.8525	-0.0295	0.8554	-0.0324
798	0.9240	0.9503	-0.0263	0.9463	-0.0223
900	0.9700	0.9869	-0.0169	0.9830	-0.0130
1002	0.9920	0.9971	-0.0051	0.9949	-0.0029
1201	1.0000	0.9999	-0.0001	0.9996	-0.0004

*Highest difference

with

$$\bar{x} = \frac{\alpha}{\beta} \tag{3a}$$

$$s^2 = \frac{\alpha}{\beta^2} \tag{3b}$$

and

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1}e^{-x}dx \tag{3c}$$

where

α,β = parameters of the gamma distribution,
$\bar{x}$ = mean of the sample,
s^2 = variance of the sample,
x = a random variable, and
Γ = the gamma function.

A computer program including the preceding formulas was used to generate the gamma distribution function. The values of alpha and beta used in this program were calculated by using Equations 3a and 3b, as follows:

$$\bar{x} = 559.3$$

$$s^2 = 18125.8$$

$$\beta = \frac{\bar{x}}{s_2} = \frac{559.3}{18125.8} = 0.0309$$

$$\alpha = s^2\beta^2 = 18125.8 \times (0.0309)^2 = 17.258$$

After substituting the values of alpha and beta into Equation 2, the final equation becomes:

$$P(x) = 1.97 \times 10^{-40} (x + c)^{16.258}e^{-0.0309(x+c)} \tag{4}$$

where

X = bus arrival time and
c = constant, the earliest bus arrival time (-522 in this situation).

The theoretical gamma cumulative distribution function and the distribution of observed data were plotted in Figure 4. The probability of bus arrival at a bus stop can then be predicted by using Equation 4.

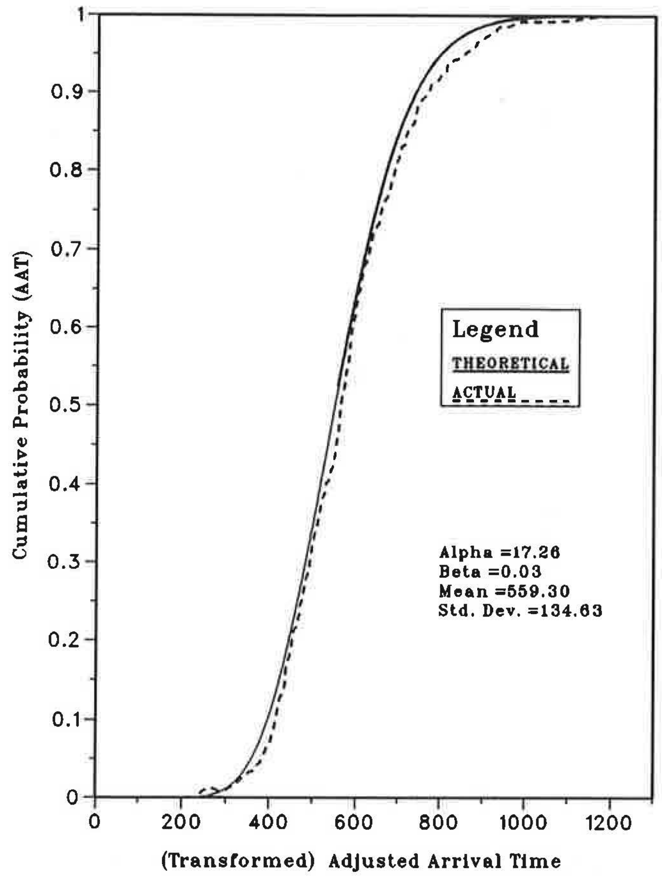

FIGURE 4 Cumulative gamma distribution of adjusted arrival times.

COMPARISONS WITH PREVIOUS STUDIES

Because past studies have used different approaches or methods, an exact comparison is difficult. Each of these studies showed different results.

Turnquist (*2*) suggested that the distribution of bus arrival times at a point along a route is log normal (see Figure 5a). Because a logarithm is not defined for zero or negative numbers, a transformation also needs to be applied to this function.

Talley and Becker (*6*) proposed that the exponential probability distribution (see Figure 5b) be used to compute the probability that buses will be more than *x* minutes early and more than *y* minutes late.

In the present study, the bus arrival times tended to follow the gamma distribution (see Figure 5c), which also requires a transformation. The existence of a shorter left tail than right tail of the gamma distribution makes it appropriate for this situation. The left tail indicates that there are some rather extreme values. This is often the case in day-to-day bus operations; some buses arrive quite early. The shorter left tail is expected because earliness, represented by the left tail, can be controlled. When drivers feel that they are early, they can slow down or wait for a time at bus stops. If they are late, however, the only thing they can do is speed up a little or, maybe, not stop at certain points along the route. Lateness can be caused by many factors that bus operators cannot control. One of them is bad traffic conditions. Because of these uncontrolled factors, some buses may arrive extremely late. These extremely late arrivals are represented by the long right tail of this gamma distribution.

CONCLUSIONS

On-time performance can be used to improve operating efficiency (better headway planning) and to increase revenue (attract more people to ride buses). In addition, passenger waiting times can be reduced. In general, on-time performance can significantly improve the public perception of the transit system.

The term *adjusted arrival time* was used for on-time performance. An analysis was made to obtain the statistics of the adjusted arrival times. The adjusted arrival times in terms

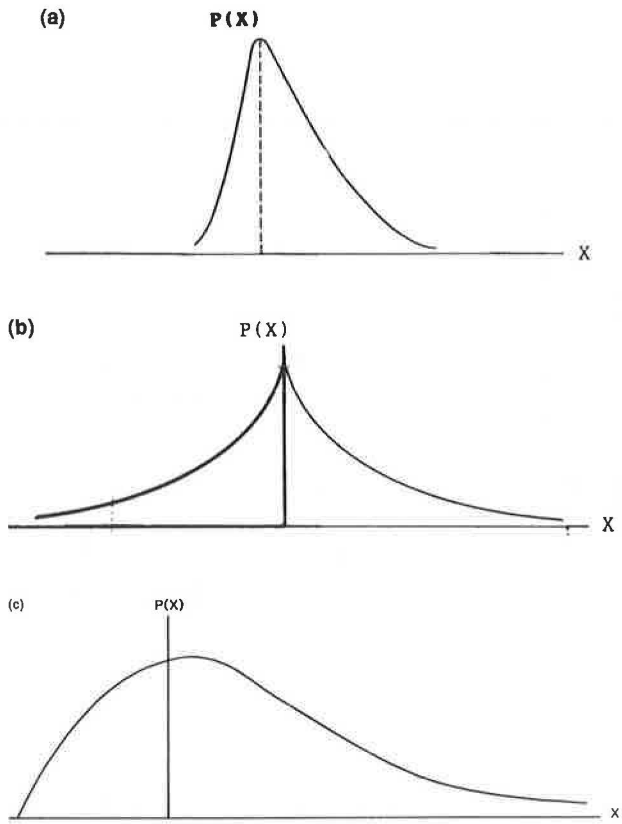

FIGURE 5 (a) Log normal distribution, (b) exponential distribution, and (c) gamma distribution.

of headway, direction, and time of day were studied. An analysis was performed to study the distribution of adjusted arrival times.

The results of the study showed that adjusted bus arrival times follow a gamma distribution. This distribution can be used to (1) measure the on-time performance using a significantly smaller sample size, (2) estimate the probability of a bus being on time, and (3) model passenger waiting times, passenger arrivals, and on-time performance. This finding differs from the ones that had been proposed in the past.

In addition, a theory was proposed that adjusted arrival times are functions of the distance along a route, the location of peak load point, and the headway. Buses in the morning and evening peaks tend to arrive late. However, the midday buses tend to arrive early.

More research is needed to improve bus transit on-time performance and to study the effect of on-time performance on ridership and on reputation of bus operators. To confirm that the gamma distribution fits bus arrival times, more data from other cities are needed. A log normal distribution should also be considered because of the similarities between the two distributions. More data are also needed to explore the theories of bus arrival times as a function of distance along a route and bus headway and why they occur as they do.

ACKNOWLEDGMENTS

Funding for this research was provided by the National Science Foundation. The cooperation of Samuel Seward of the Milwaukee County Transit System is acknowledged. The opinions expressed in this paper are those of the authors, not of the National Science Foundation or the Milwaukee County Transit System. The assistance of Darcie De Young is also appreciated.

REFERENCES

1. J. W. Bates. *Transportation Research Circular 300: Definition of Practices for Bus Transit On-Time Performance: Preliminary Study.* TRB, National Research Council, Washington, D.C., 1986.
2. M. A. Turnquist. A Model for Investigating the Effects of Service Frequency and Reliability on Bus Passenger Waiting Times. In *Transportation Research Record 663*, TRB, National Research Council, Washington, D.C., 1978, pp. 70–73.
3. R. M. Shanteau. Estimating the Contribution of Various Factors to Variations in Bus Passenger Loads at a Point. In *Transportation Research Record 789*, TRB, National Research Council, Washington, D.C., 1981, pp. 8–11.
4. M. Abkowitz and J. Tozzi. Research Contributions to Managing Transit Service Reliability. *Journal of Advanced Transportation*, Vol. 21, No. 1, Spring 1987, pp. 47–65.
5. R. P. Guenthner and K. C. Sinha. Maintenance, Schedule Reliability, and Transit System Performance. *Transportation Research*, Vol. 17A, No. 5, September 1983, pp. 355–362.
6. W. K. Talley and A. J. Becker. On-Time Performance and the Exponential Probability Distribution. In *Transportation Research Record 1108*, TRB, National Research Council, Washington, D.C., 1987, pp. 22–26.
7. IMSLS Inc. *IMSL User's Manual.* Houston, Texas 77036-5085, 1984.
8. D. L. Gerlough and F. C. Barnes. *Poisson and Other Distributions in Traffic.* Eno Foundation for Transportation, Saugatuck, Conn., 1971.

Publication of this paper sponsored by Committee on Bus Transit Systems.